TRIG EQUATION REVIEW WORKSHEET KEY

1. a) $\sin x = 0.67$

$x = \sin^{-1}(0.67)$

$x = \boxed{0.73}$ $\quad x = \pi - 0.73 = \boxed{2.41}$

b) $4\sin x + 5 = 0$

$4\sin x = -5$

$\sin x = -5/4$

$\boxed{\text{no solution}}$

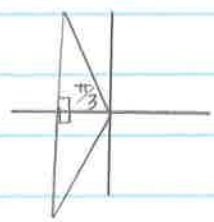
c) $4\sec x + 8 = 0$

$4\sec x = -8$

$\sec x = \frac{-8}{4}$

$\sec x = -2$

$\frac{1}{\cos x} = -2$



$\cos x = -1/2$

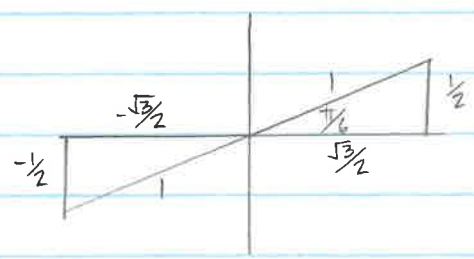
$x = \pi - \pi/3 = \boxed{2\pi/3}$

$x = \pi + \pi/3 = \boxed{4\pi/3}$

d) $\cot x - \sqrt{3} = 0$

$\cot x = \sqrt{3}$

$\frac{1}{\tan x} = \sqrt{3}$



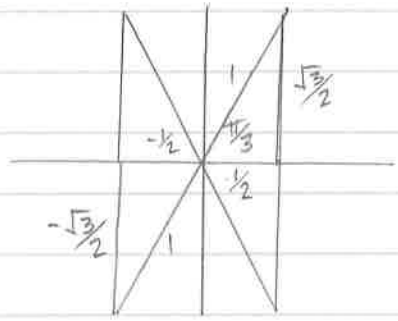
$\tan x = \frac{1}{\sqrt{3}}$ $\quad x = \boxed{\pi/6}$ $\quad x = \pi + \pi/6 = \boxed{7\pi/6}$

2 a) $3\cot^2 x - 1 = 0$

$3\cot^2 x = 1$

$\cot^2 x = \frac{1}{3}$

$\cot x = \pm\sqrt{1/3} = \pm\frac{1}{\sqrt{3}}$



$\tan x = \pm\sqrt{3}$

$x = \boxed{\pi/3}$ $\qquad$ $x = \pi + \pi/3 = \boxed{4\pi/3}$

$x = \pi - \pi/3 = \boxed{2\pi/3}$ $\qquad$ $x = 2\pi - \pi/3 = \boxed{5\pi/3}$

b) $2\sin^2 x + 5\sin x = 3$

$2\sin^2 x + 5\sin x - 3 = 0$

$(2\sin x - 1)(\sin x + 3) = 0$

$2\sin x - 1 = 0$ or $\sin x + 3 = 0$

$2\sin x = 1$ $\qquad$ $\sin x = -3$

$\sin x = 1/2$ $\qquad$ no solution

$\boxed{x = \pi/6,\ 5\pi/6}$

c) $2\tan^2 x - \tan x - 6 = 0$

$(2\tan x + 3)(\tan x - 2) = 0$

$2\tan x + 3 = 0$ or $\tan x - 2 = 0$

$2\tan x = -3$ $\qquad$ $\tan x = 2$

$\tan x = -3/2$ $\qquad$ $x = \tan^{-1}(2)$

$x = \tan^{-1}(-3/2)$ $\qquad$ ② $\boxed{x = 1.11}$

① $x = -0.9828$ ← reference angle

tan is neg. in QUAD II, IV

① $x = \pi - 0.9828 = \boxed{2.16}$

$x = 2\pi - 0.9828 = \boxed{5.30}$

② $x = \pi + 1.11 = \boxed{4.25}$

2 d) $\sec x \sin x - 3\sin x = 0$

$\sin x(\sec x - 3) = 0$

$\sin x = 0$ or $\sec x - 3 = 0$

$\boxed{x = 0, \pi}$

$\frac{1}{\cos x} - 3 = 0$

$\frac{1}{\cos x} = 3$

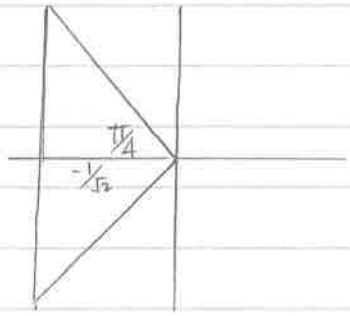
$\cos x = 1/3$

$x = \cos^{-1}(1/3) = \boxed{1.23}$

cosine is positive in QUAD I and IV

$x = 2\pi - 1.23 = \boxed{5.05}$

3 ai) $\cos 2x = -1/\sqrt{2}$



let $2x = A$

$\therefore \cos A = -1/\sqrt{2}$

$A = \pi - \pi/4 = 3\pi/4$

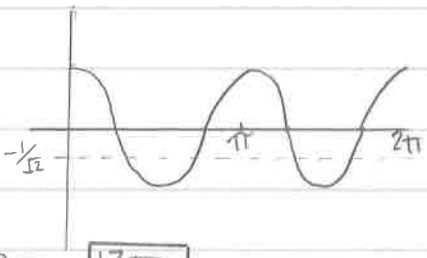
$A \quad \pi + \pi/4 = 5\pi/4$

period $= 2\pi/2 = \pi$

but, $2x = A$

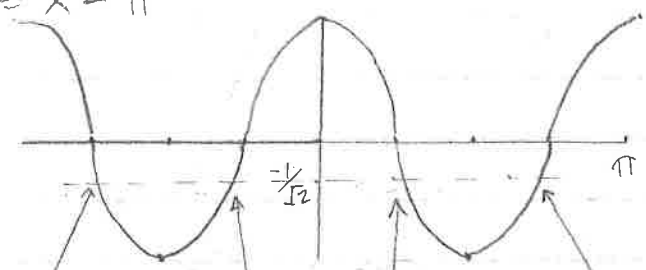
$\therefore \quad 2x = 3\pi/4$ and $2x = 5\pi/4$

$x = \boxed{3\pi/8}$ $\quad x = \boxed{5\pi/8}$



$x = 3\pi/8 + \pi = 3\pi/8 + 8\pi/8 = \boxed{11\pi/8}$ $\quad x = 5\pi/8 + 8\pi/8 = \boxed{13\pi/8}$ $\quad 0 \le x < 2\pi$

aii) if $-\pi \le x < \pi$



$x = \frac{3\pi}{8} - \pi = \boxed{-\frac{5\pi}{8}}$

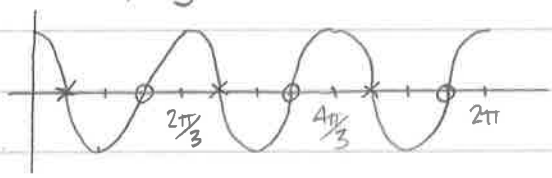
$x = \boxed{\frac{3\pi}{8}}$

$x = \boxed{\frac{5\pi}{8}}$

$x = \frac{5\pi}{8} - \pi = \boxed{-\frac{3\pi}{8}}$

3 b) (i) $\cos 3x = 0$

period $= 2\pi/3$



let $3x = A$

$\therefore \cos A = 0$

$A = \pi/2, \ 3\pi/2$

but $3x = A$ $\quad \therefore 3x = \pi/2$ and $3x = 3\pi/2$

$x = \boxed{\pi/6}$ × $\qquad x = 3\pi/6 = \boxed{\pi/2}$ ○

$x = \pi/6 + 2\pi/3 = \pi/6 + 4\pi/6 = \boxed{5\pi/6}$ ×

$x = 5\pi/6 + 4\pi/6 = 9\pi/6 = \boxed{3\pi/2}$ ×

$x = 3\pi/6 + 4\pi/6 = \boxed{7\pi/6}$ ○

$x = 7\pi/6 + 4\pi/6 = \boxed{\frac{11\pi}{6}}$ ○ $\qquad 0 \le x \le 2\pi$

(ii) general solution

$$\boxed{\begin{aligned} x &= \pi/6 + 2\pi n/3 \\ x &= \pi/2 + 2\pi n/3 \end{aligned}}$$

where n is an integer

3. c) $\cos^2 x - \cos x - 2 = 0$

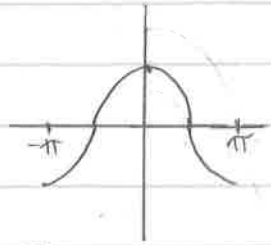
$(\cos x - 2)(\cos x + 1) = 0$

$\cos x - 2 = 0$ or $\cos x + 1 = 0$

$\cos x = -2$ $\quad$ $\cos x = -1$

no solution $\quad$ $x = \pi$

(i) for domain $-\pi \leq x < \pi$ $\quad$ $\boxed{x = -\pi}$



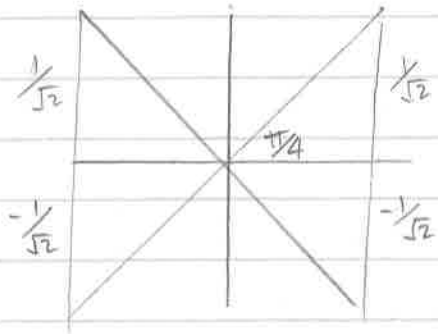
ii) for domain $-\pi/2 \leq x < \pi/2$ $\quad$ $\boxed{\text{no solution}}$

4. $2\sin^2 x - 1 = 0$

$2\sin^2 x = 1$

$\sin^2 x = 1/2$

$\sin x = \frac{1}{\pm\sqrt{2}}$



$x = \pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4$ $\quad$ difference of $\pi/2$ between solutions

general solution

$x = \pi/4 + n\pi/2$ $\quad$ where n is an integer

5. a) $\cos 5x = 0$, horiz. comp. factor $1/5$ $\therefore$ 10 solutions

b) $\sin 4x = 1$ $\quad$ $\sin x = 1$ only has one solution $0 \leq x \leq 2\pi$

$\therefore$ 4 solutions

c) 6 solutions for $2\sin 3x - 1 = 0$

2 solutions for $\cos 2x + 1 = 0$ $\quad$ $\therefore$ 8 solutions

6. $\tan 5x = 3.4$ period $= \frac{\pi}{5}$

let $A = 5x$

$\therefore \tan A = 3.4$

$A = \tan^{-1}(3.4)$

$A = 1.28$

but $A = 5x$

$\therefore 5x = 1.28$

$x = 0.26$

$\boxed{x = 0.26 + \frac{n\pi}{5}, \text{ where } n \text{ is an integer}}$

7 OMIT, these questions require a graphing calculator