SOLUTIONS

BASI

14 Prove the identity: **(5 marks)**

$$(\csc\theta - \sin\theta)\tan\theta = \frac{\sin 2\theta}{2\sin\theta}$$

LEFT SIDE	RIGHT SIDE
$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\frac{\sin\theta}{\cos\theta}$	$= \frac{2\sin\theta\cos\theta}{2\sin\theta}$
$= \left(\frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta}\right)\frac{\sin\theta}{\cos\theta}$	$= \cos\theta$
$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\frac{\sin\theta}{\cos\theta}$	
$= \frac{\cos^2\theta}{\cos\theta}$	
$= \cos\theta$	
L.S. = R.S. Q.E.D.	

15 Prove the identity: **(5 marks)**

$$\frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x} = 2\tan^2 x$$

LEFT SIDE	RIGHT SIDE
$= \frac{\sin x}{1-\sin x} \times \frac{1+\sin x}{1+\sin x} - \frac{\sin x}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}$	
$= \frac{\sin x + \sin^2 x}{1-\sin^2 x} - \frac{\sin x - \sin^2 x}{1-\sin^2 x}$	
$= \frac{\sin x + \sin^2 x - (\sin x - \sin^2 x)}{1-\sin^2 x}$	
$= \frac{\sin x + \sin^2 x - \sin x + \sin^2 x}{\cos^2 x}$	
$= \frac{2\sin^2 x}{\cos^2 x} = 2\tan^2 x$	
L.S. = R.S. Q.E.D.	

16 Prove: **(5 marks)**

$$\frac{\cot\theta}{\sin\theta-\csc\theta} = -\sec\theta$$

LEFT SIDE	RIGHT SIDE
$= \dfrac{\frac{\cos\theta}{\sin\theta}}{\sin\theta-\frac{1}{\sin\theta}}$	
$= \dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{\sin^2\theta}{\sin\theta}-\frac{1}{\sin\theta}}$	
$= \dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{\sin^2\theta-1}{\sin\theta}}$	
$= \dfrac{\cos\theta}{\cancel{\sin\theta}} \times \dfrac{\cancel{\sin\theta}}{\sin^2\theta-1}$	
$= \dfrac{\cos\theta}{-\cos^2\theta} = \dfrac{1}{-\cos\theta} = -\sec\theta$	L.S. = R.S. Q.E.D.

17 Prove the identity: **(5 marks)**

$$\frac{\cos\theta+\cot\theta}{1+\sin\theta} = \cot\theta$$

LEFT SIDE	RIGHT SIDE
$\dfrac{\cos\theta+\frac{\cos\theta}{\sin\theta}}{1+\sin\theta} = \dfrac{\frac{\sin\theta\cos\theta}{\sin\theta}+\frac{\cos\theta}{\sin\theta}}{1+\sin\theta}$	
$= \dfrac{\frac{\sin\theta\cos\theta+\cos\theta}{\sin\theta}}{1+\sin\theta}$	
$= \dfrac{\cos\theta\,(\cancel{\sin\theta+1})}{\sin\theta\,\cancel{(1+\sin\theta)}}$	
$\dfrac{\cos\theta}{\sin\theta} = \cot\theta$	
L.S. = R.S. Q.E.D.	

18 Prove the identity: **(4 marks)**

$$\frac{\cos x + \cot x}{\sec x + \tan x} = \cos x \cot x$$

LEFT SIDE	RIGHT SIDE
$\dfrac{\cos x + \frac{\cos x}{\sin x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} = \dfrac{\frac{\sin x \cos x}{\sin x} + \frac{\cos x}{\sin x}}{\frac{1+\sin x}{\cos x}}$	
$= \dfrac{\frac{\sin x \cos x + \cos x}{\sin x}}{\frac{1+\sin x}{\cos x}} = \dfrac{\frac{\cos x(\sin x + 1)}{\sin x}}{\frac{1+\sin x}{\cos x}}$	
$= \dfrac{\cos x (\sin x + 1)}{\sin x} \times \dfrac{\cos x}{1+\sin x}$	
$= \dfrac{\cos x \cos x}{\sin x} = \cot x \cos x$	
L.S. = R.S.	
Q.E.D.	

19 Prove the identity: **(4 marks)**

$$\tan\theta \cos 2\theta + \tan\theta = \sin 2\theta$$

factor out $\tan\theta$

LEFT SIDE	RIGHT SIDE
$= \tan\theta(\cos 2\theta + 1)$	$\cos 2\theta = 2\cos^2\theta - 1$
$= \tan\theta(2\cos^2\theta - 1 + 1)$	
$= \dfrac{\sin\theta}{\cos\theta}(2\cos^2\theta)$	
$= \sin\theta(2\cos\theta)$	
$= 2\sin\theta\cos\theta$	
$= \sin 2\theta$	
L.S. = R.S.	
Q.E.D.	

20. Prove the identity: **(4 marks)**

$$\csc\theta\sin 2\theta - \sec\theta\cos 2\theta = \sec\theta$$

LEFT SIDE

$$\frac{1}{\sin\theta} \cdot 2\sin\theta\cos\theta - \frac{1}{\cos\theta}(2\cos^2\theta - 1)$$

$$= 2\cos\theta - \frac{(2\cos^2\theta - 1)}{\cos\theta}$$

$$= \frac{2\cos^2\theta}{\cos\theta} - \frac{(2\cos^2\theta - 1)}{\cos\theta}$$

$$= \frac{2\cos^2\theta - 2\cos^2\theta + 1}{\cos\theta}$$

$$= \frac{1}{\cos\theta} = \sec\theta$$

RIGHT SIDE

L.S. = R.S.

Q.E.D.

21 Prove the identity: **(4 marks)**

$$\frac{1-\cos 2x}{\sin 2x} = \frac{1+\tan x}{1+\cot x}$$

LEFT SIDE

$$= \frac{1-(1-2\sin^2 x)}{2\sin x\cos x}$$

$$= \frac{1-1+2\sin^2 x}{2\sin x\cos x}$$

$$= \frac{2\sin^2 x}{2\sin x\cos x}$$

$$= \frac{\sin x}{\cos x}$$

RIGHT SIDE

$$= \frac{1+\frac{\sin x}{\cos x}}{1+\frac{\cos x}{\sin x}}$$

$$= \frac{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}{\frac{\sin x}{\sin x}+\frac{\cos x}{\sin x}}$$

$$= \frac{\frac{\cos x+\sin x}{\cos x}}{\frac{\sin x+\cos x}{\sin x}}$$

$$= \frac{\cos x+\sin x}{\cos x} \times \frac{\sin x}{\sin x+\cos x} = \frac{\sin x}{\cos x}$$

L.S. = R.S. Q.E.D.

22 Prove the identity. **(5 marks)**

$$\cos 2x = \frac{\cot x - \sin 2x}{\cot x}$$

LEFT SIDE	RIGHT SIDE
$= 1 - 2\sin^2 x$	$= \dfrac{\frac{\cos x}{\sin x} - 2\sin x \cos x}{\frac{\cos x}{\sin x}}$
	$= \dfrac{\frac{\cos x}{\sin x} - \frac{2\sin^2 x \cos x}{\sin x}}{\frac{\cos x}{\sin x}}$
	$= \dfrac{\frac{\cos x - 2\sin^2 x \cos x}{\cancel{\sin x}}}{\frac{\cos x}{\cancel{\sin x}}}$
	$= \dfrac{\cos x (1 - 2\sin^2 x)}{\cos x}$
	$= 1 - 2\sin^2 x$ L.S. = R.S. Q.E.D.

23 Prove the identity: **(5 marks)**

$$\frac{\tan x(\cos x + \cot x)}{\sec x + \tan x} = \frac{\sin x \sin 2x}{2 - 2\cos^2 x}$$

LEFT SIDE	RIGHT SIDE
$= \dfrac{\frac{\sin x}{\cos x}\left(\cos x + \frac{\cos x}{\sin x}\right)}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}$	$= \dfrac{\sin x (2\sin x \cos x)}{2(1 - \cos^2 x)}$
$= \dfrac{\frac{\sin x \cancel{\cos x}}{\cancel{\cos x}} + \frac{\cancel{\sin x}\cos x}{\cos x \cancel{\sin x}}}{\frac{1 + \sin x}{\cos x}}$	$= \dfrac{2\sin^2 x \cos x}{2\sin^2 x}$
	$= \cos x$
$= \dfrac{\frac{\sin x + 1}{\cos x}}{\frac{1 + \sin x}{\cos x}} = \cos x$	L.S. = R.S. Q.E.D.

24 Prove the identity: **(5 marks)**

$$\frac{\tan x}{\sec x + 1} = \frac{2\cos x - 2\cos^2 x}{\sin 2x}$$

LEFT SIDE	RIGHT SIDE
$= \dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + 1}$	$= \dfrac{2\cos x(1-\cos x)}{2\sin x \cos x}$
$= \dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\cos x}{\cos x}}$ find a common denominator	$= \dfrac{1-\cos x}{\sin x}$
$= \dfrac{\frac{\sin x}{\cos x}}{\frac{1+\cos x}{\cos x}}$ rewrite as a single fraction	

$= \dfrac{\sin x}{\cancel{\cos x}} \times \dfrac{\cancel{\cos x}}{1+\cos x} = \dfrac{\sin x}{1+\cos x}$

multiply by conjugate

$= \dfrac{\sin x}{1+\cos x} \times \dfrac{(1-\cos x)}{(1-\cos x)}$

$= \dfrac{\sin x(1-\cos x)}{1-\cos^2 x}$ ← Pythagorean identity

$= \dfrac{\sin x(1-\cos x)}{\sin^2 x}$

$= \dfrac{1-\cos x}{\sin x}$

L.S. = R.S.

Q.E.D.