PRE–CALCULUS 12 **MOCK FINAL EXAM**

PART A: MULTIPLE CHOICE (non-calculator)
SECTION I

Suggested Time: 35 minutes
Allowable Time: 45 minutes

INSTRUCTIONS: No calculator may be used for this section of the examination. For each question, select the **best** answer and record your choice on the bubble sheet provided. Using an HB pencil, completely fill in the bubble [ ] that has the letter corresponding to your answer.

1. Given y = -4 cos (3x - π) –1, determine the following:

   a) amplitude 4

   b) phase shift $\pi/3$ units to the right

   $-4\cos 3\left(x-\frac{\pi}{3}\right)-1$

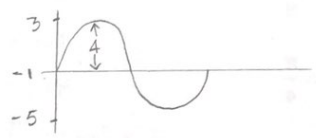
   c) vertical displacement 1 unit down

   d) period $2\pi/3$

   e) maximum value 3

   f) minimum value -5

   g) range $-5 \le y \le 3$



2. Which expression is equivalent to $4 - 6\cos^2 12x$?

   A. 1 + 3cos6x

   B. 1 + 3cos24x

   C. 1 – 3cos6x

   D. 1 – 3cos24x

$= -6\cos^2 12x + 4$
$= -(6\cos^2 12x - 4)$
$= -(6\cos^2 12x - 3 - 1)$
$= -(3\cos 24x - 1)$
$= -3\cos 24x + 1$
$= 1 - 3\cos 24x$

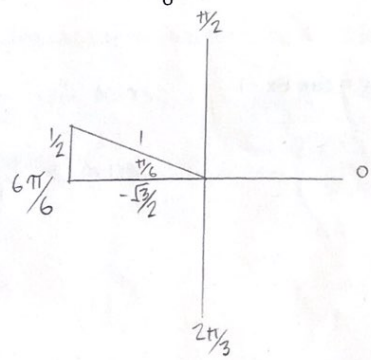
3. Determine the exact value of $\sec\frac{5\pi}{6}$.

   A. $-\frac{\sqrt{3}}{2}$

   B. $-\frac{2}{\sqrt{3}}$

   C. $\frac{2}{\sqrt{3}}$

   D. $\frac{\sqrt{3}}{2}$



$\sec\theta = \frac{H}{A}$
$= \frac{1}{-\sqrt{3}/2}$
$= -\frac{2}{\sqrt{3}}$

4. Solve $2\sin^2 x - 7\sin x - 4 = 0$, where $0 \le x \le 2\pi$.

A. $\frac{\pi}{6}, \frac{5\pi}{6}$

B. $0, \frac{\pi}{6}, \frac{5\pi}{6}$

C. $\frac{7\pi}{6}, \frac{11\pi}{6}$

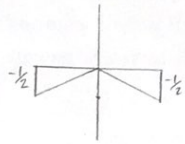
D. $0, \frac{7\pi}{6}, \frac{11\pi}{6}$

$(2\sin x + 1)(\sin x - 4) = 0$

$2\sin x + 1 = 0$ or $\sin x - 4 = 0$

$2\sin x = -1$ $\quad$ $\sin x = 4$

$\sin x = -\frac{1}{2}$ $\quad$ no solution



5. Which of the following is an asymptote of the function $y = \csc 4x$?

A. $x = \frac{\pi}{16}$

B. $x = \frac{\pi}{8}$

C. $x = \frac{\pi}{4}$

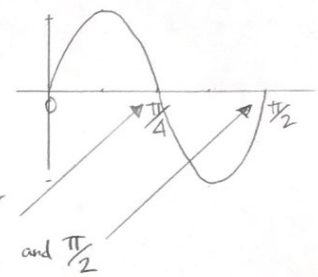
D. $x = \frac{\pi}{2}$

$\csc 4x = \frac{1}{\sin 4x}$

$\sin 4x$ looks like

period $= \frac{2\pi}{4} = \frac{\pi}{2}$

$\therefore$ the asymptote for $\frac{1}{\sin 4x}$ is @ $\frac{\pi}{4}$ and $\frac{\pi}{2}$



6. Determine an expression equivalent to $\sin(2x - \pi)$.

A. $2\sin x \cos x$

B. $-2\sin x \cos x$

C. $\cos^2 x - \sin^2 x$

D. $\sin^2 x - \cos^2 x$

$= \sin 2x \cos\pi - \cos 2x \sin\pi$

$= \sin 2x(-1) - \cos 2x(0)$

$= -\sin 2x$

$= -2\sin x \cos x$

7. Determine the period of $y = \tan 6x$.

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

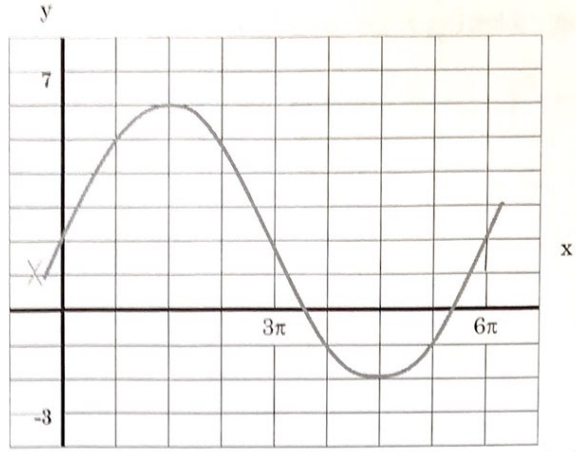
C. π

D. 2π

period for $y = \tan bx$ is π/b

$\therefore$ period $= \pi/6$

8. If the graph below has the equation $y = a\sin bx + d$, determine the value of b ($b > 0$).



A. $\frac{1}{3}$

B. 3

C. 2π

D. 6π

period of graph is 6π

$\therefore \frac{2\pi}{b} = 6\pi$

$\frac{2\cancel{\pi}}{6\cancel{\pi}} = b$

$b = \frac{1}{3}$

9. If cot x = 0, then which of the following must be true?

A. $\sin x = 0, \cos x = 0$

B. $\sin x = 0, \cos x \neq 0$

C. $\sin x \neq 0, \cos x = 0$

D. $\sin x \neq 0, \cos x \neq 0$

$\cot x = \frac{\cos x}{\sin x}$

10. Determine an equivalent expression for 3 log A – 2 logB – log C.

A. $\log\left(\frac{A^3}{B^2C}\right)$

B. $\log\left(\frac{A^3C}{B^2}\right)$

C. $\log(A^3 - B^2 - C)$

D. $\log(3A - 2B - C)$

$= \log A^3 - \log B^2 - \log C$

$= \log\left(\frac{A^3}{B^2}\right) - \log C$

$= \log\left(\frac{\frac{A^3}{B^2}}{C}\right)$

$= \log\left(\frac{A^3}{B^2} \div C\right)$

$= \log\left(\frac{A^3}{B^2} \times \frac{1}{C}\right)$

11. Evaluate: $2\log_b\left(\frac{1}{b^3}\right)$ $= 2\log_b (b)^{-3}$

$= -6\log_b b$

$= -6$

A. -6

B. $-\frac{2}{3}$

C. $\frac{2}{3}$

D. 6

12. A population, P_o, of bacteria doubles every 5 hours. This colony grows to a final population, P, after t hours. Which equation could be used to determine the value of t?

A. $P_o = P(2)^{\frac{t}{5}}$

B. $P_o = P(2)^{\frac{5}{t}}$

C. $P = P_o(2)^{\frac{t}{5}}$

D. $P = P_o(2)^{\frac{5}{t}}$

13. If $\log 5 = k$, determine an expression for $\log\left(\frac{1}{2}\right)$

$\log(1/2) = \log(5/10)$

$= \log 5 - \log 10$

$= k - 1$

A. $\frac{k}{10}$

B. $-k$

C. $k - 1$

D. $10 - k$

14. Evaluate $\log_2 \frac{1}{8}$ $= \log_2\left(\frac{1}{2^3}\right) = \log_2 2^{-3} = -3\log_2 2 = -3$

A. -4

B. -3

C. $\frac{1}{4}$

D. $\frac{1}{3}$

15. Which equation represents the graph of $y = f(x - 1) + 3$ after the graph is translated 4 units to the right.

A. $y = f(x - 1) - 1$

B. $y = f(x - 1) + 7$

C. $y = f(x - 5) + 3$

D. $y = f(x + 3) + 3$

replace x with x-4

$\therefore\ y = f((x-4)-1)+3$

$y = f(x-5)+3$

16. omitted

This is the end of Part A, Section I

You may proceed to the rest of the examination *without* the use of a calculator until directed by the supervisor to access your calculator. At the end of 45 minutes, you will not be able to go back to Part A, Section I; therefore, ensure you have checked this section.

PART A: MULTIPLE CHOICE (calculator permitted)
SECTION II

Suggested Time: 55 minutes

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the bubble sheet provided. Using an HB pencil, completely fill in the bubble [ ] that has the letter corresponding to your answer.

17. If $f(x) = x + 2$ and $g(x) = x^2 + 1$, what is $(g \circ f)(x)$?

A. $x^2 + 3$

B. $x^2 + x + 3$

C. $x^2 + 4x + 5$

D. $x^2 + 4x + 1$

$= g(f(x))$
$= g(x+2)$
$= (x+2)^2 + 1$
$= x^2 + 4x + 4 + 1$

18. The y-intercept of the function $y = f(x)$ is 5. Determine the y-intercept of $y = -f(x) + 3$.

①

A. -2

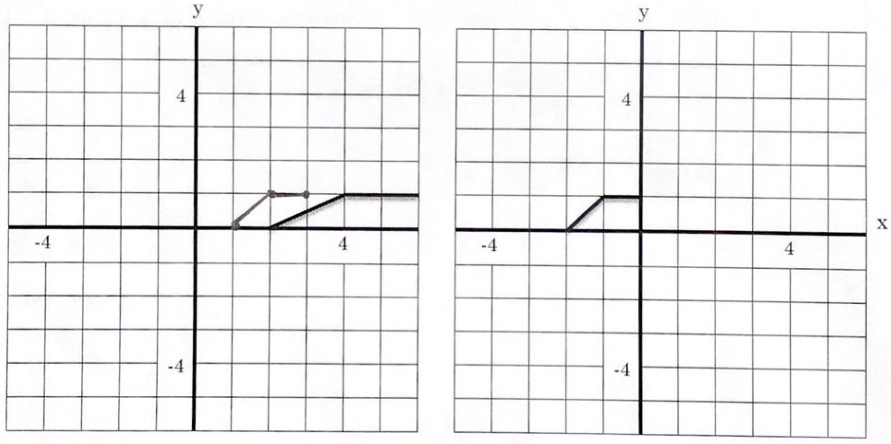
B. -8

C. 8

D. 2

y-int is @ $(0,5)$
① replace y with -y ∴ $(0,-5)$
② vert. trans. 3 units up ∴ $(0,-2)$

19. The graph of $y = f(x)$ is shown below on the left. Determine an equation of the function graphed on the right.



A. $y = f(2x + 3)$

B. $y = f(2x + 6)$

C. $y = 2f(x + 3)$

D. $y = 2f(x + 6)$

horiz. comp. factor ½ $y = f(2x)$
horiz. trans 3 units LEFT
∴ $y = f(2(x+3))$
$y = f(2x+6)$

20. Determine the inverse of the function $f(x) = \frac{x}{x+1}$

A. $f^{-1}(x) = \frac{x+1}{x}$

B. $f^{-1}(x) = \frac{-x}{x+1}$

C. $f^{-1}(x) = \frac{-x}{x-1}$

D. $f^{-1}(x) = \frac{-1}{x-1}$

$y = \frac{x}{x+1}$

$x = \frac{y}{y+1}$

$(y+1)x = y$

$xy + x = y$

$x = y - xy$

$x = y(1-x)$

$\frac{x}{1-x} = y$

$f^{-}(x) = \frac{x}{1-x}$

$= \frac{x}{-(-1+x)}$

$= \frac{-x}{x-1}$

21. Let θ be an angle in standard position such that $\cot\theta = \frac{-4}{3}$ and $\sin\theta < 0$. Determine the exact value of sec θ.

quadrants II or IV

quadrant III or IV

A. $-\frac{5}{3}$

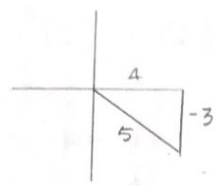
B. $-\frac{5}{4}$

C. $\frac{5}{4}$

D. $\frac{5}{3}$

∴ we are in quad. IV

$\cot\theta = \frac{A}{O}$



hypotenuse $= \sqrt{4^2 + (-3)^2} = 5$

∴ $\sec\theta = \frac{H}{A} = \frac{5}{4}$

22. A wheel rolling along the ground has a diameter of 16 cm and rotates every 12 seconds. At time $t = 0$ s, a point P on the outside edge of the wheel is at its highest point. Determine the cosine function that gives the height, h, of point P above the ground at any time t, where h is in cm and t is in seconds.

A. $h(t) = -8\cos\frac{\pi}{6}t + 8$

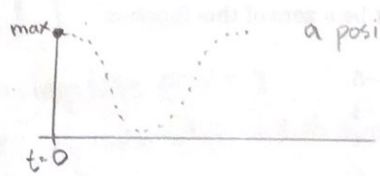
B. $h(t) = 8\cos\frac{\pi}{12}t + 8$

C. $h(t) = 8\cos\frac{\pi}{6}t + 8$

D. $h(t) = -8\cos\frac{\pi}{12}t + 8$

A = radius = 8 cm

period $= \frac{2\pi}{12} = \frac{\pi}{6}$



a positive cosine functio

23. A circle has a radius of 12 cm. If the central angle subtended by two radii is 45°. Determine the arc length.

A. 2π cm

B. 3π cm

C. 4π cm

D. 6π cm

$a = r\theta$

$a = 12\left(\frac{\pi}{4}\right)$

$a = 3\pi$ cm

$\frac{\pi}{4}$ radians

24. Determine the number of solutions for $(a\sin x - b)(a\cos x - a)(b\sin x + a) = 0$ where $0 \le x < 2\pi$, if $0 < a < b$.

A. 3

B. 4

C. 5

D. 6

since the product = 0

$a\sin x - b = 0$ or $a\cos x - a = 0$ or $b\sin x + a = 0$

$a\sin x = b$ | $a\cos x = a$ | $b\sin x = -a$

$\sin x = b/a$ | $\cos x = a/a$ | $\sin x = -a/b$

but, $0<a<b$ | $\cos x = 1$

$\therefore$ no solution | $\therefore$ 1 solution | $\therefore$ 2 solutions

25. Determine the equation of the polynomial, in factored form, if the zeros are 1, 3 and the y-intercept is 6.

A. $y = \frac{1}{2}(x+1)(x+3)$

B. $y = 2(x+1)(x+3)$

C. $y = \frac{1}{2}(x-1)(x-3)$

D. $y = 2(x-1)(x-3)$

$y = a(x-1)(x-3)$

passing thru $(0,6)$

$\therefore$ $6 = a(0-1)(0-3)$

$6 = a(-1)(-3)$

$6 = 3a$

$a = 2$

26. Determine the remainder when x^3-1 is divided by $x+2$.

A. −7

B. −9

C. 0

D. 3

$f(-2) = (-2)^3 - 1$

$= -8 - 1$

$= -9$

27. When $x^3 + kx + 1$ is divided by $x-2$, the remainder is −3. Find the value of k.

A. −12

B. −6

C. −5

D. 4

$f(2) = 2^3 + 2k + 1 = -3$

$8 + 2k + 1 = -3$

$9 + 2k = -3$

$2k = -3 - 9$

$2k = -12$

$k = -6$

28. If $P(x)$ is a polynomial function where $P(-3) = 5$, then which of the following **could not** be a zero of this function?

A. −5

B. −3

C. 3

D. 5

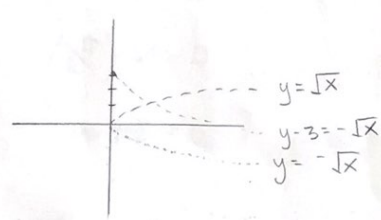
29. Determine the range of $y - 3 = -\sqrt{2x-4}$

A. $y \ge 2$

B. $y \le 3$

C. $y \ge -2$

D. $y \ge 3$



30. What are the vertical and horizontal asymptotes of the rational function below?

$$\frac{x^2+3x-1}{4-x^2} = \frac{x^2+3x-1}{(2+x)(2-x)}$$

A. Vertical: $x = 2$; Horizontal: $y = -1$

B. Vertical: $x = 2$; Horizontal: $y = 0$

C. Vertical: $x = \pm 2$; Horizontal: $y = -1$

D. Vertical: $x = \pm 2$; Horizontal: $y = 0$

- vert. asymptote @ $x = \pm 2$
- degrees are = ∴ horiz. asymptote is at the ratio of the coefficients

$\frac{1}{-1} = -1$

31. Which of the following graphs *does not* have a horizontal asymptote?

A. $f(x) = \frac{2x^2-3x+5}{x+2}$

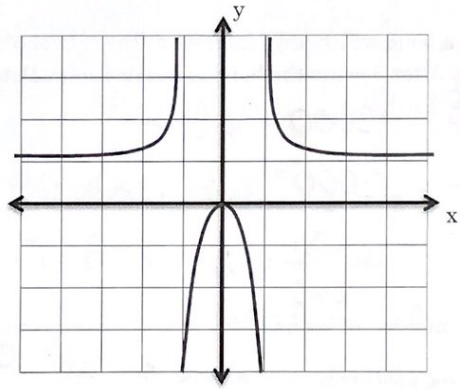
B. $f(x) = \frac{3x}{x^2-4}$

C. $f(x) = \frac{x^2-4}{x^2+2}$

D. $f(x) = \frac{1}{x+3}$

look for the function where the degree of numerator > denominator

32. Determine the equation of the function below.



A. $\frac{1}{x^2}$

B. $\frac{-x^2}{x^2-1}$

C. $\frac{x^2}{x^2-1}$

D. $\frac{4}{x^2+1}$

① horiz. asymptote @ $x = 1$

∴ degree of numerator and denominator are equal

② vert. asymptotes @ $x = -1$ and $x = 1$

33. Evaluate: $2\log_b(\frac{1}{b^3})$

A. -6

B. $-\frac{2}{3}$

C. $\frac{2}{3}$

D. 6

same as question 11

34. Sue purchases a car for \$5000. The value of the car depreciates at a rate of 8% per year. What will be the value of the car at the end of 10 years?

A. \$4600.00

B. \$2171.94

C. \$ 400.00

D. \$1000.00

$V = 5000(0.92)^{10} = 2171.94$

35. An earthquake in Vancouver measured 3.5 on the Richter Scale. Another earthquake near Turkey was 2000 times as intense. What was the Richter Scale reading for the earthquake near Turkey?

A. 3.3

B. 5.5

C. 6.8

D. 7.3

$\frac{10^x}{10^{3.5}} = 2000 \longrightarrow 10^{x-3.5} = 2000$

$\log 2000 = x - 3.5 \longrightarrow \log 2000 + 3.5 = x$

$x = 6.8$

36. Sue invests in a bond which pays interest at the rate of 8% per annum compounded semi-annually. After 3 years the bond is worth \$2000. Determine the initial value of the bond.

A. \$1260.34

B. \$1580.63

C. \$1587.66

D. \$1777.99

$2000 = V_0(1.04)^{3(2)}$

$2000 = V_0(1.265)$

$\frac{2000}{1.265} = V_0$

37. Determine the number of terms in the geometric sequence:

$\frac{1}{81}, \frac{1}{27}, \frac{1}{9}, \ldots, 531441 \longrightarrow$ use $t_n = ar^{n-1}$ ① where: $t_n = 531441$, $a = \frac{1}{81}$, $r = \frac{1/27}{1/81} = \frac{81}{27} = 3$

A. 14

B. 15

C. 16

D. 17

② $\therefore 531441 = \frac{1}{81}(3)^{n-1}$

$43046721 = 3^{n-1} \longrightarrow$ ③ $\log_3 43046721 = n-1$

④ $\frac{\log 43046721}{\log 3} + 1 = n$

38. Evaluate: $\sum_{k=3}^{13} 5(-2)^{k-1}$ $r = -2$

$n = \#$ of terms $= 13 - 3 + 1 = 11$

A. 13 660

B. 13 655

C. −13 651.67

D. −13 646.67

$a = 5(-2)^{3-1} = 5(-2)^2 = 20$

$l = 5(-2)^{13-1} = 5(-2)^{12} = 20480$

$S_n = \frac{a - rl}{1 - r}$

$S_n = \frac{20 - (-2)20480}{1 - (-2)}$

$= 13660$

$a=5,\ d=-4,\ t_n=-111$

39. Determine the sum of the arithmetic series $5 + 1 + (-3) + \ldots + (-111)$

A. – 1594
B. – 1590
C. – 1586
D. – 1582

① determine n, the number of terms.

$t_n = a + (n-1)d$

$-111 = 5 + (n-1)(-4)$

$-111 = 5 - 4n + 4$

$-111 = 9 - 4n$

$-120 = -4n$

$n = 30$

② $S_n = \frac{n}{2}(a+l)$

$= \frac{30}{2}(5+(-111))$

$= -1590$

40. Determine the 1st term of a geometric sequence if the common ratio is 3 and the 5th term is 162.

$t_5 = a(3)^{5-1}$

A. – 3
B. – 2
C. 2
D. 3

$162 = a(3)^4$

$162 = a \cdot 81 \longrightarrow \frac{162}{81} = a \longrightarrow a = 2$

41. If $-2x$, -4, x^2 are three consecutive terms in a geometric sequence, determine the value of x.

$\frac{-4}{-2x} = \frac{x^2}{-4} \longrightarrow 16 = -2x^3 \longrightarrow \frac{16}{-2} = x^3$

A. – 2
B. ± 2
C. ± 4
D. Ø

$-8 = x^3 \longrightarrow x = -2$

42. For a certain geometric series, $S_n = 2 - 2(-3)^n$. Determine t_3

A. 48
B. 60
C. 72
D. 84

$t_1 = S_1 = 2 - 2(-3)^1 = 2 + 6 = 8$

$S_2 = 2 - 2(-3)^2 = 2 - 2(9)$

$= 2 - 18 = -16$

this means that $t_1 + t_2 = -16$

but $t_1 = 8$ $\therefore 8 + t_2 = -16$

$t_2 = -16 - 8 = -24$

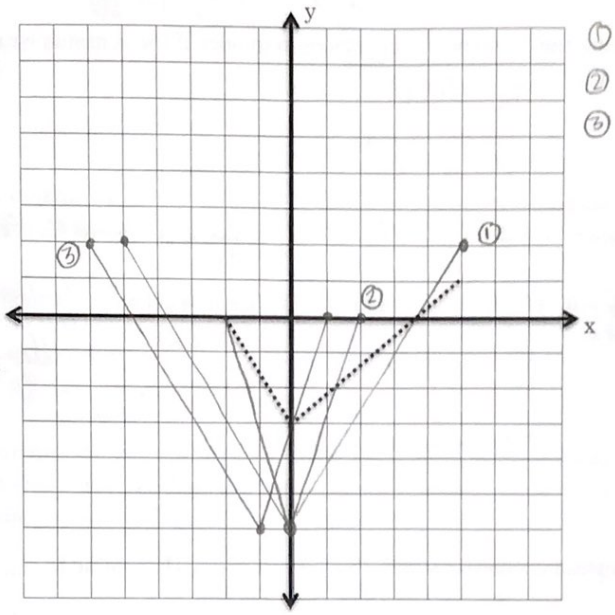
$\therefore$ the sequence looks like $8, -24, t_3$

$r = \frac{-24}{8} = -3$

$\therefore t_3 = -24 \times -3 = 72$

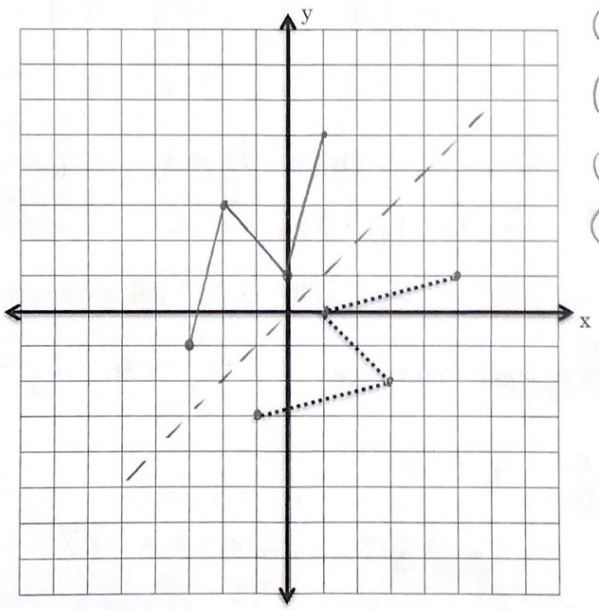
PART B: WRITTEN RESPONSE (calculator permitted) / 32

1. Given the graph of y = f (x), sketch the graph of $y = 2f-(x+1)$ on the grid below.
① ② ③ (2 marks)



① vert. exp. factor 2
② ref. in y-axis
③ horiz. trans 1 unit LEFT

2. Given the graph of y = *f* (x), sketch the graph of the inverse on the grid below. (2 marks)



$(-1,-3) \rightarrow (-3,-1)$

$(3,-2) \rightarrow (-2,3)$

$(1,0) \rightarrow (0,1)$

$(5,1) \rightarrow (1,5)$

3. Solve algebraically using logarithms to at least 2 decimal places.

$$2^x = 5^{x+1}$$ (5 marks)

$\log 2^x = \log 5^{x+1}$

$x \log 2 = (x+1)\log 5$

$x\log 2 = x\log 5 + \log 5$

$x\log 2 - x\log 5 = \log 5$

$x(\log 2 - \log 5) = \log 5$

$x = \dfrac{\log 5}{\log 2 - \log 5}$ $\boxed{x = -1.76}$

check:

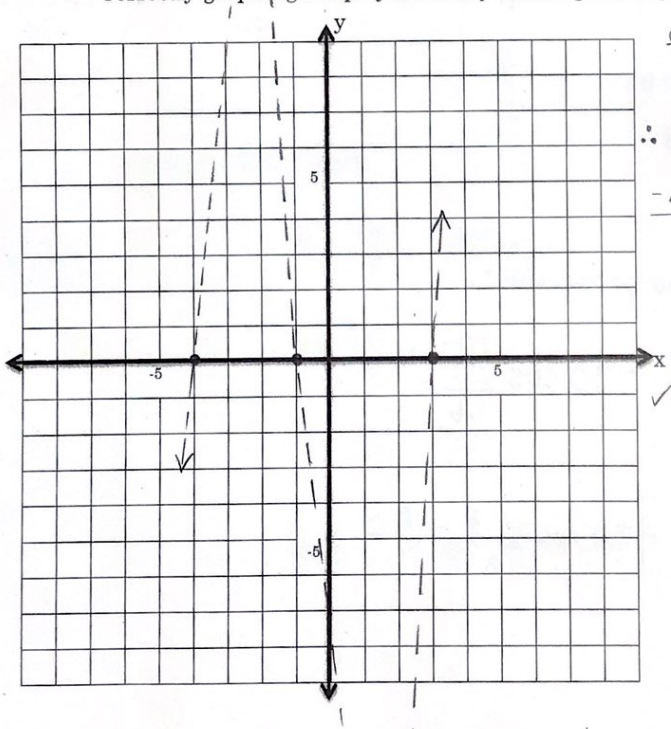
$2^{-1.76} = 0.296$

$5^{-1.76+1} = 0.296$ ✓

4. Graph: $f(x) = x^3 + 2x^2 - 11x - 12$ (4 marks)

Marks will be rewarded for:

- Using the Factor Theorem to obtain at least one zero
- Listing the zeros and stating their multiplicity
- Correctly graphing the zeros, y-intercept and local maximums and minimums
- Correctly graphing the polynomial by showing the degree, and end behaviour.



$f(-4) = -4^3 + 2(-4)^2 - 11(-4) - 12$

$= -64 + 32 + 44 - 12 = 0$ ✓

$\therefore (x+4)$ is a factor

-4	1	2	-11	-12
	↓	-4	8	12
	1	-2	-3	0

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

min @ $x = 1.4$ (approx)

$y = -20$

max @ $x = -2.5$ (approx)

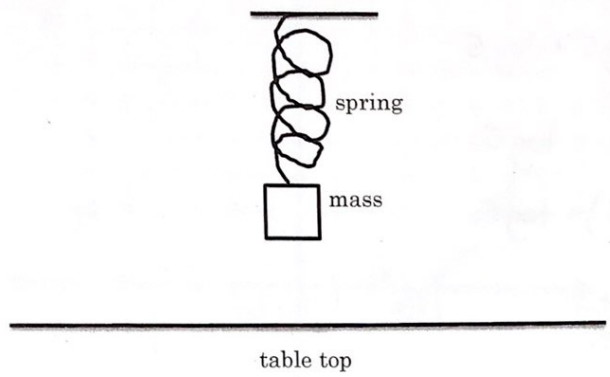
$y = 12$ ✓

zeros @ $x = -4$ multiplicity 1

$x = -1$ 1 ✓

$x = 3$ 1

5. A mass is supported by a spring so that it rests 50 cm above a table top, as shown in the diagram below. The mass is pulled down to a height of 20 cm above the table top and released at time = 0. It take 0.8 seconds for the mass to reach a maximum height of 80 cm above the table top. As the mass moves up and down, its height h, in cm, above the table top, is approximated by a sinusoidal function of the elapsed time t, in seconds, for a short period of time.



Determine an equation for a sinusoidal function that gives h as a function of t. (4 marks)

① mid-line of graph @ $h = 50$

② minimum = 20

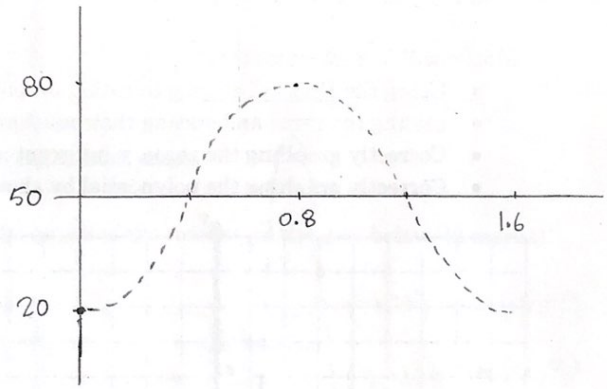
$\therefore$ amplitude = 30

and maximum = 80

③ from min $\longrightarrow$ max takes 0.8 s

this is half of the period

$\therefore$ period = 1.6



The graph shows no phase shift.

$$\therefore \quad h = -30\cos\frac{2\pi}{1.6}t + 50$$

or $$h = -30\cos\frac{\pi}{0.8}t + 50$$

6. Solve algebraically over the set of real numbers: (4 marks)

$$2\cos^2 x + 3\cos x + 1 = 0$$

period $= 2\pi$

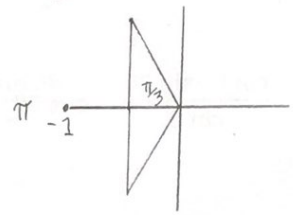
State the general solution using **exact values** if possible.
Otherwise, answer to **at least 2 decimal places**.

factor this $\longrightarrow$ $(2\cos x + 1)(\cos x + 1) = 0$

$\therefore$ $2\cos x + 1 = 0$ or $\cos x + 1 = 0$

$2\cos x = -1$ $\qquad$ $\cos x = -1$

$\cos x = -\frac{1}{2}$ $\qquad$ $x = \pi$



$x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

General Solution

$x = \frac{2\pi}{3} + 2\pi n$

$x = \frac{4\pi}{3} + 2\pi n$

$x = \pi + 2\pi n$

where n is an integer

7. Solve (3 marks)

$$-x - 1 + \sqrt{1+x} = 0$$

$-x - 1 + \sqrt{1+x} = 0$

$\sqrt{1+x} = x+1$

$1+x = (x+1)^2$

$1+x = (x+1)(x+1)$

$1+x = x^2+2x+1$

$0 = x^2+2x+1-x-1$

$0 = x^2+x$

$0 = x(x+1)$

$\boxed{x=0}$ or $\boxed{x=-1}$

CHECK:

if $x=0$

$-1+\sqrt{1} = 0$ ✓

if $x=-1$

$-(-1)-1+\sqrt{1+(-1)} \stackrel{?}{=} 0$

$1-1+\sqrt{1-1} \stackrel{?}{=} 0$

$0+\sqrt{0} = 0$ ✓

8. Prove the identity: (5 marks)

$$\frac{\cot\theta - \sin 2\theta}{\cot\theta} = \cos 2\theta$$

LEFT SIDE	RIGHT SIDE
$= \dfrac{\frac{\cos\theta}{\sin\theta} - 2\sin\theta\cos\theta}{\frac{\cos\theta}{\sin\theta}}$	
$= \dfrac{\frac{\cos\theta}{\sin\theta} - 2\sin\theta\cos\theta \times \frac{\sin\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}}$	
$= \dfrac{\frac{\cos\theta - 2\sin^2\theta\cos\theta}{\cancel{\sin\theta}}}{\frac{\cos\theta}{\cancel{\sin\theta}}}$	
$= \dfrac{\cos\theta(1-2\sin^2\theta)}{\cos\theta}$	
$= 1-2\sin^2\theta$	
$= \cos 2\theta$	L.S. = R.S.
	Q.E.D.