Pre-Calculus 12 | Trigonometric Equations Quiz | Name: KEY

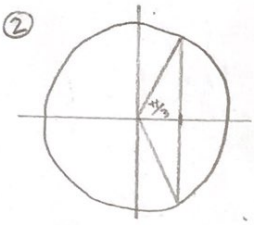
Non-Calculator Questions

1. Solve exactly: $\cos \frac{1}{2}x = 0.5$ where $0 \le x < 2\pi$

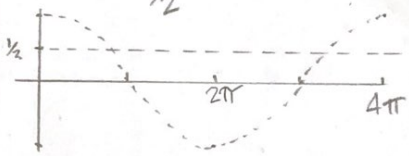
① let $\frac{1}{2}x = A$

$\therefore \cos A = \frac{1}{2}$

$A = \frac{\pi}{3}$ $\quad A = \frac{5\pi}{3}$

② 

④ Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$



③ but $\frac{1}{2}x = A$

$\therefore \frac{1}{2}x = \frac{\pi}{3}$ $\quad \frac{1}{2}x = \frac{5\pi}{3}$

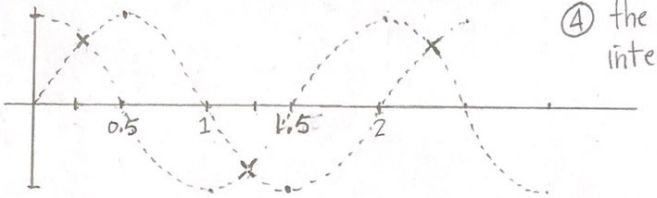
⑤ $\boxed{x = \frac{2\pi}{3}}$ $\quad x = \frac{10\pi}{3}$ outside of domain

2. Find the general solution to $\sin \pi x = \cos \pi x$. In the answers, n is an integer.

A. $0.25 + n\pi$
B. $0.25 + n$
C. $0.25 - n\pi$
D. $0.25 + 2n\pi$
E. $0.25 - 2n\pi$

① period $= \frac{2\pi}{\pi} = 2$

② draw both graphs on the same grid



③ graphs intersect at $x = 0.25$ and $x = 1.25$, $x = 2.25$ etc...

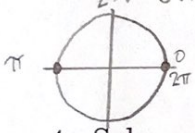
④ the difference between each intersection is 1

3. What is the general solution to $x \sin 4x = 0$? In the answers, n is an integer.

① $\boxed{x = 0}$ or $\sin 4x = 0$

let $4x = A$

$\therefore \sin A = 0$

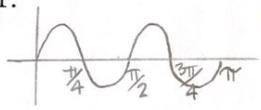


$A = 0, \pi, 2\pi$

② but, $4x = A$

$\therefore 4x = 0 \quad 4x = \pi \quad 4x = 2\pi$

$x = 0 \quad x = \frac{\pi}{4} \quad x = \frac{2\pi}{4}$

③ 

④ $\boxed{x = \frac{n\pi}{4}, \text{ where } n \text{ is an integer}}$

4. Solve exactly: $\cos 3x = 0$, where $0 \le x < 2\pi$

① let $3x = A$

$\cos A = 0$

$A = \frac{\pi}{2} \quad A = \frac{3\pi}{2}$

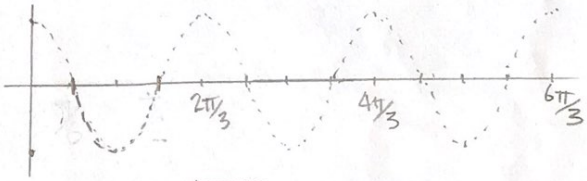
but $3x = A$

$\therefore 3x = \frac{\pi}{2} \quad 3x = \frac{3\pi}{2}$

$\boxed{x = \frac{\pi}{6}} \quad x = \frac{3\pi}{6} = \boxed{\frac{\pi}{2}}$

② period $= \frac{2\pi}{3}$

$= \frac{4\pi}{6}$



③ $x = \frac{\pi}{6} + \frac{2\pi}{3} = \boxed{\frac{5\pi}{6}}$ $\quad x = \frac{5\pi}{6} + \frac{4\pi}{6} = \frac{9\pi}{6} = \boxed{\frac{3\pi}{2}}$

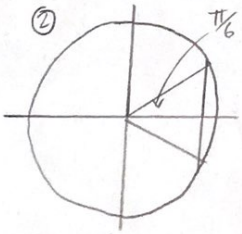
$x = \frac{\pi}{2} + \frac{2\pi}{3} = \boxed{\frac{7\pi}{6}}$ $\quad x = \frac{7\pi}{6} + \frac{4\pi}{6} = \boxed{\frac{11\pi}{6}}$

5. Solve exactly: $\cos x = \sqrt{3} - \cos x$, where $0 \le x < 2\pi$

① $\cos x + \cos x = \sqrt{3}$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

② 

$\therefore \boxed{x = \frac{\pi}{6}}$

$x = 2\pi - \frac{\pi}{6} = \boxed{\frac{11\pi}{6}}$

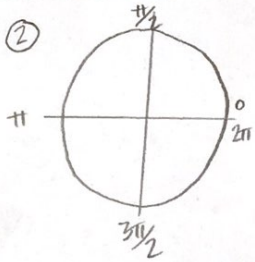
6. Solve exactly: $\sin^2 x - \sin x = 0$, where $0 \le x < 2\pi$

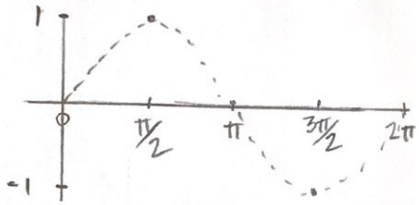
① $\sin x(\sin x - 1) = 0$

$\sin x = 0$ or $\sin x - 1 = 0$

③ $\boxed{x = 0, \pi}$ $\sin x = 1$

$\boxed{x = \pi/2}$

② 



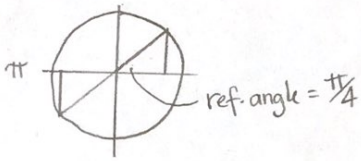
7. Solve exactly: $\tan^2 x = \tan x$, where $0 \le x < 2\pi$

$\tan^2 x - \tan x = 0$

① $\tan x(\tan x - 1) = 0$

$\tan x = 0$ or $\tan x - 1 = 0$

③ $\boxed{x = 0, \pi}$ $\tan x = 1$ → $\boxed{x = \pi/4, 5\pi/4}$

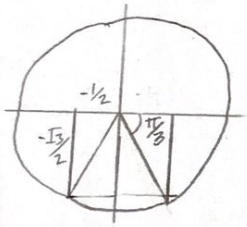
② 

$\pi + \pi/4$

8. $\sin \frac{1}{2}x = -\frac{\sqrt{3}}{2}$ where $0 \le x < 2\pi$

① let $\frac{1}{2}x = A$

$\sin A = -\sqrt{3}/2$

② 

③ $A = \pi + \pi/3$ $A = 2\pi - \pi/3$

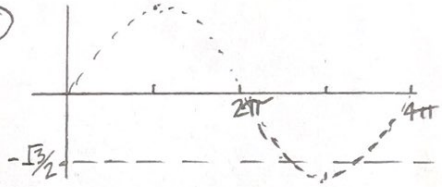
$= 4\pi/3$ $= 5\pi/3$

④ but $\frac{1}{2}x = A$ $\frac{1}{2}x = 5\pi/3$

$\therefore \frac{1}{2}x = 4\pi/3$ $x = 10\pi/3$

$x = 8\pi/3$

period = 4π

⑤ 

$\boxed{\text{NO SOLUTION}}$

Calculator Question

9. Solve to two decimal places: $\sin x = 0.34$ where $0 \le x < 2\pi$
(Hint: There are two answers.)

$x = \sin^{-1}(0.34)$

$\boxed{x = 0.35}$ but, sin is also positive in quadrant II

0.35 is a reference angle!!

$\therefore \boxed{x = \pi - 0.35 = 2.79}$