Math 12 Final Exam Pre Test/Exam Outline

Name ☺ KEY

NO CALC

1. Determine the range, maximum and minimum values of the following graph

$$y = -3\cos 2(x+3) - 1$$

amplitude = 3

vert. disp. = -1

$\boxed{R: -4 \le y \le 2}$

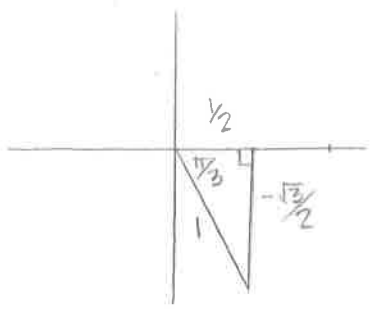
$\boxed{\begin{array}{l} max = 2 \\ min = -4 \end{array}}$

2. Simplify the following sinx+cosxcotx

$$\sin x + \cos x \frac{\cos x}{\sin x} = \frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x} = \frac{1}{\sin x} = \boxed{\csc x}$$

3. Determine the exact value :

$$\csc\frac{5\pi}{3} = \frac{H}{O} = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}}$$



$$\frac{-2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{-2\sqrt{3}}{3}}$$

4. Solve

$$2\cos^2 x + \cos x - 3 = 0$$

$(2\cos x + 3)(\cos x - 1) = 0 \longrightarrow$

$2\cos x + 3 = 0$

$2\cos x = -3$

$\cos x = -3/2$

$x = \cos^{-1}(3/2)$

no solution

or

$\cos x - 1 = 0$

$\cos x = 1$

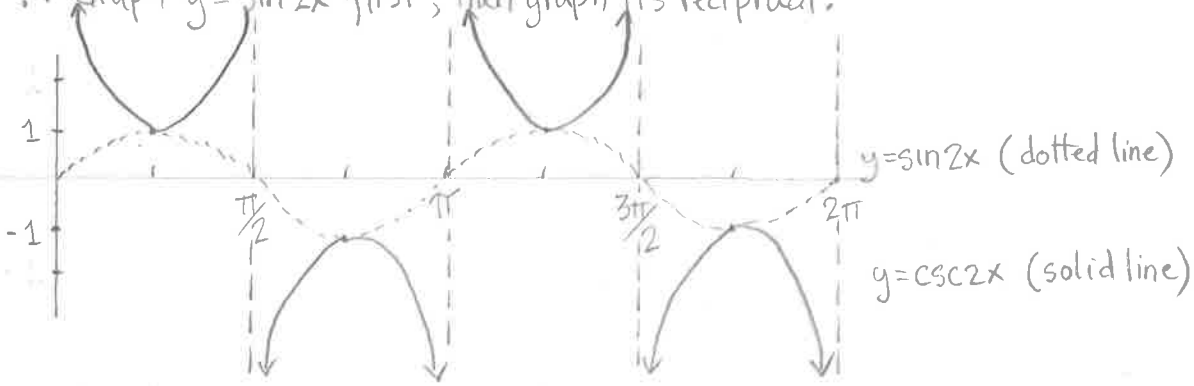
$\boxed{x = 0}$

5. Sketch a graph of the graph of y = csc2x

The graph of $y = \csc 2x$ is the same as $y = \frac{1}{\sin 2x}$

the reciprocal of $y = \sin 2x$, Period $= \frac{2\pi}{2} = \pi$

∴ Graph $y = \sin 2x$ first, then graph it's reciprocal.



6. Determine an expression equivalent to $14\cos^2 12x - 7$

use $\cos 2\theta = 2\cos^2\theta - 1$

- in this case, $14\cos^2 12x - 7 = 7(2\cos^2 12x - 1)$

$= 7\cos 2(12x) = \boxed{7\cos 24x}$

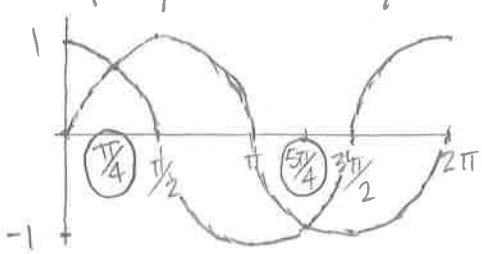
7. Determine the period of the graph y=5sin3x

period $= \frac{2\pi}{b}$ for $y = \sin bx$ and $y = \cos bx$

$b = 3 \quad \therefore \quad \text{period} = \boxed{\frac{2\pi}{3}}$

8. Solve cosx=sinx, where $0 \le x < 2\pi$.

Graph $y = \sin x$ and $y = \cos x$ on same grid. Intersection points represent the solution.



$\boxed{x = \pi/4,\ 5\pi/4}$

9. What is the asymptote for the graph $y = 2\log_3(x-1) + 3$

* $x - 1$ must be greater than zero

$x - 1 > 0 \longrightarrow x > 1 \longrightarrow$ asymptote is @ $\boxed{x = 1}$

10. Determine an equivalent expression to 2logx - logy – 5logz

$= \log x^2 - [\log y + \log z^5]$

$= \log x^2 - [\log yz^5]$

$\boxed{= \log\left(\frac{x^2}{yz^5}\right)}$

11. Determine an equivalent expression to $\frac{\log a}{\log b}$

use change of base rule

$$\boxed{= \log_b a}$$

12. In a Petri dish bacteria triples every 3 hours. If there were 300 bacteria to start and there are currently 5000 bacteria, determine the equation that would be used to find how long the bacteria have been in the dish.

triples every 3 hours $\xrightarrow{\text{means}}$ $3^{t/3}$

$\therefore \quad 5000 = 300(3)^{t/3}$

$\frac{5000}{300} = 3^{t/3}$

take the log of both sides

$\log\left(\frac{50}{3}\right) = \frac{t}{3}\log 3$

$$\boxed{\frac{3\log\left(\frac{50}{3}\right)}{\log 3} = t}$$

13. Solve: $5(25^{2x+1}) = \left(\frac{1}{125}\right)^{3x}$

$5(5^2)^{2x+1} = (5^{-3})^{3x}$

$5^1(5^{4x+2}) = 5^{-9x}$

$5^{1+4x+2} = 5^{-9x}$

since the bases are =, the exponents must be =

$\therefore \quad 4x+3 = -9x$

$3 = -9x - 4x$

$3 = -13x$

$$\boxed{x = -\frac{3}{13}}$$

14. If log a = 2.5 and log b = 1.5 find $\log\frac{1000a^2}{b}$

$\log\frac{1000a^2}{b} = \log 1000 + \log a^2 - \log b$

$= 3 + 2\log a - 1.5$ (because $\log b = 1.5$)

$= 3 + 2(2.5) - 1.5$ (because $\log a = 2.5$)

$= 3 + 5 - 1.5$

$= \boxed{6.5}$

15. Simplify $\dfrac{(x-2)!(x+1)!}{(x!)^2}$

$$= \frac{(x-2)!\,(x+1)(x)(x-1)(x-2)!}{\left[x(x-1)(x-2)!\right]^2} = \frac{\cancel{(x-2)!}\,(x+1)\cancel{(x)}\cancel{(x-1)}\cancel{(x-2)!}}{x(x-1)\cancel{(x-2)!}\;\cancel{x(x-1)}\cancel{(x-2)!}}$$

$$= \frac{x+1}{x(x-1)} = \boxed{\frac{x+1}{x^2-x}}$$

16. If the graph of $y = 2f(x) - 3$ is reflected over the y-axis and then translated 5 units left, what is the new equation?

① replace x with −x

② replace x with x+5

① $y = 2f(-x) - 3$

② $y = 2f(-(x+5)) - 3$

$$\boxed{y = 2f(-x-5) - 3}$$

Calculators allowed

17. The graph of $y = f(x)$ is reflected over the x-axis and vertically compressed by a factor of ½. Determine the equation of the new graph.

① replace y with −y

② replace y with 2y

① $-y = f(x)$

② $-2y = f(x)$

$$\boxed{y = -\tfrac{1}{2}f(x)}$$

18. Describe what happens to the graph of $y = f(x)$ if it becomes $y = -2f(3x+9) - 5$.

rewrite as $y = -2f(3(x+3)) - 5$

vertical translation 5 units DOWN

reflection about x-axis

horizontal compression factor ⅓

horizontal translation 3 units LEFT

vertical expansion by a factor of 2

19. Find the inverse of f(x) if $f(x)=\frac{2x+1}{5x}$. $\xrightarrow[\text{as}]{\text{rewrite}}$ $y=\frac{2x+1}{5x}$

switch x and y

$x=\frac{2y+1}{5y}$ $\xrightarrow{\text{solve for } y}$ $5xy=2y+1 \longrightarrow 5xy-2y=1 \longrightarrow y(5x-2)=1$

$y=\frac{1}{5x-2} \longrightarrow \boxed{f^{-1}(x)=\frac{1}{5x-2}}$

20. If the point (-3,9) is on the graph of y=f(x). Determine the coordinates of the point after the transformation given $y=3f(-\frac{1}{3}x)+1$

① ② ③ ④

① vert. exp. fact. 3 $\longrightarrow$ (-3,27)

② ref. in y-axis $\longrightarrow$ (3,27)

③ horiz. exp. fact. 3 $\longrightarrow$ (9,27)

④ vert. trans. 1 unit up $\longrightarrow$ $\boxed{(9,28)}$

21.. If the point (2,-4) is on the graph of y=f(x). Determine the coordinates of the point after the transformation given $y+1=f(-\frac{1}{2}x+1)$

rewrite as $y+1=f(-\frac{1}{2}(x-2))$

④ ① ② ③

① ref. in y-axis $\longrightarrow$ (-2,-4)

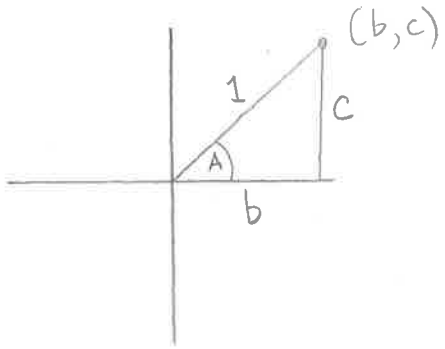
② horiz. exp. fact. 2 $\longrightarrow$ (-4,-4)

③ horiz. trans. 2 units RIGHT $\longrightarrow$ (-2,-4)

④ vert. trans. 1 unit DOWN $\longrightarrow$ $\boxed{(-2,-5)}$

*translations are always done LAST

22. The terminal arm of angle A in standard position intersects the unit circle at the point (b,c). Determine an expression that would represent 5cscA.



$\csc\theta=\frac{\text{hyp}}{\text{opp}}$

$\therefore\ 5\csc A=\frac{5(1)}{c}=\boxed{\frac{5}{c}}$

23. In a circle, an arc length 10.5 cm contains a central angle of 22°. Determine the radius of the circle.

use $a = r\theta$ ——— remember, θ must be in radians

$$\frac{180^\circ}{22^\circ} = \frac{\pi}{\theta} \longrightarrow \theta = \frac{\pi \times 22^\circ}{180^\circ} = 0.384 \text{ radians}$$

$$r = \frac{a}{\theta} \longrightarrow r = \frac{10.5\text{ cm}}{0.384} \longrightarrow \boxed{r = 27.35\text{ cm}}$$

24. Determine the restriction(s) for the expression $\dfrac{\tan x - 2}{\csc x}$

$$\frac{\tan x - 2}{\csc x} = \frac{\frac{\sin x}{\cos x} - 2}{\frac{1}{\sin x}}$$

$$\boxed{\begin{array}{l} \cos x \neq 0 \\ \csc x \neq 0 \\ \sin x \neq 0 \end{array}}$$

25. Determine the equation of the polynomial in factored form if the zeros are -2,-1,2 and the y-intercept is -5.

$y = a(x+2)(x+1)(x-2)$

passing through $(0,-5)$

$\therefore\ -5 = a(0+2)(0+1)(0-2)$

$-5 = a(2)(1)(-2)$

$-5 = -4a$

$\frac{-5}{-4} = a$

$a = \frac{5}{4}$

$$\boxed{y = \tfrac{5}{4}(x+2)(x+1)(x-2)}$$

26. Determine the reminder when $x^5 + 2x^4 - 3x^3 - 2x^2 + 5x - 1$ is divided by x-1.

$P(1) = 1^5 + 2(1)^4 - 3(1)^3 - 2(1)^2 + 5(1) - 1$

$= 1 + 2 - 3 - 2 + 5 - 1$

$= \boxed{2}$

OR SYNTHETIC DIVISION

1	1	2	-3	-2	5	-1
	↓	1	3	0	-2	3
	1	3	0	-2	3	2

REMAINDER

27. If a polynomial P(x) is divided by x-3, what is the remainder?

$$\boxed{P(3)}$$

28. When $3x^3 - x^2 + kx + 5$ is divided by x+1, the reminder is 7. Find the value of k.

$P(-1) = 3(-1)^3 - (-1)^2 + k(-1) + 5 = 7$

$-3 - 1 - k + 5 = 7$

$-4 - k + 5 = 7$

$1 = 7 + k$

$k = 1 - 7$

$\boxed{k = -6}$

29. Determine the domain of the following function: $y - 5 = -\sqrt{3x - 9}$

$3x - 9$ MUST be greater than or equal to zero in order take the square root of it.

$\therefore$ $3x - 9 \geq 0$

$3x \geq 9 \longrightarrow \therefore$ $\boxed{x \geq 3}$

30. What are the vertical and horizontal asymptotes of the following rational function: $\frac{5x^3 - 20x}{x^3 - x^2 - 6x}$?

$$\frac{5x(x^2-4)}{x(x^2-x-6)} = \frac{5x(x+2)(x-2)}{x(x+2)(x-3)} = \frac{5(x-2)}{(x-3)}$$

horizontal asymptote @ $y = 5$

vertical asymptote @ $x = 3$

31. How do I know if a rational equation does not have a horizontal asymptote?

When the top polynomial is more than 1 degree higher than the bottom polynomial.

ex. $\frac{3x^3+1}{4x+1}$

32. What is the decimal representation of $\log_5 234$ $= \frac{\log 234}{\log 5} = \boxed{3.39}$

- use the change of base rule

which states $\log_b a = \frac{\log a}{\log b}$

33. Find $(f \circ g)(x)$ if $f(x) = x^2 - 5x$ and $g(x) = x + 3$

$$f(g(x)) = f(x+3) = (x+3)^2 - 5(x+3)$$

$$= x^2 + 6x + 9 - 5x - 15$$

$$\boxed{= x^2 + x - 6}$$

34. I invested \$5000 in a bond at a rate of 4.5% per annum compounded monthly. What will the value of the bond be worth at the end of 6 years?

$$A = 5000\left(1 + \frac{.045}{12}\right)^{12n} \qquad \text{where } n = 6$$

$$\boxed{A = \$6546.52}$$

35. Determine all of the different arrangements of all of the letters of the word EQUILATERAL.

$$\frac{11!}{2!\,2!\,2!} = \boxed{4989600}$$

2 E's
2 L's
2 A's

36. Determine the number of pathways of a small town if there are 6 blocks travelling east from the north-west corner of the city and 7 blocks south to the south-east corner of the city.

1	1	1	1	1	1	1
1	2	3	4	5	6	7
1	3	6	10	15	21	28
1	4	10	20	35	56	84
1	5	15	35	70	126	210
1	6	21	56	126	252	462
1	7	28	84	210	462	924
1	8	36	120	330	792	1716

OR

$$\frac{13!}{6!\,7!} = \boxed{1716}$$

37. In a standard deck of 52 cards, how many poker hands are there with exactly 2 aces and one jack?

2 aces , 1 jack , 2 other cards

$${}_4C_2 \times {}_4C_1 \times {}_{44}C_2 = 6 \times 4 \times 946$$

$$= \boxed{22704}$$

38. In the expansion of $\left(x^2 - \frac{2}{x}\right)^6$ what is the coefficient of the constant term?

$$\left(x^2 - \tfrac{2}{x}\right)^6 = {}_6C_0 (x^2)^6 + {}_6C_1 (x^2)^5 \left(-\tfrac{2}{x}\right) + {}_6C_2 (x^2)^4 \left(-\tfrac{2}{x}\right)^2 + {}_6C_3 (x^2)^3 \left(-\tfrac{2}{x}\right)^3$$

$$+ \underbrace{{}_6C_4 (x^2)^2 \left(-\tfrac{2}{x}\right)^4} + \ldots$$

$${}_6C_4 (x^4)\left(\tfrac{16}{x^4}\right) \longrightarrow {}_6C_4 \text{ is the coefficient}$$

39. A book store has 5 new fiction books, 3 new poetry books, 7 new non-fiction books and 3 new cook books. The owner wants to display all of the books in a row in the store window, but wants to keep all of the fiction, non-fiction, poetry and cook books together. How many different arrangements are possible?

$$\frac{5!}{4} \times \frac{3!}{3} \times \frac{7!}{2} \times \frac{3!}{1} \quad \text{to arrange the 4 types}$$

$$5! \cdot 3! \times 7! \times 3! \times 4! = \boxed{522\,547\,200}$$

40. There are 7 brunettes, 5 blondes, 2 red-heads and 5 people with black hair trying out for 4 spots in a hair commercial. How many different combinations are possible if at most 2 blondes are chosen for the parts?

At most 2 blondes means <u>no blondes</u> , <u>one blonde</u> , <u>two blondes</u>

5 blondes and 14 others $\longrightarrow$ ${}_{14}C_4 + {}_5C_1 \cdot {}_{14}C_3 + {}_5C_2 \cdot {}_{14}C_2$

$$= 1001 + (5)(364) + (10)(91)$$

$$= 1001 + 1820 + 910$$

$$= \boxed{3731}$$

Trig. 9
Logs. 8 + compl interest
Transform. 6

8 written questions:

1. Trig Equation
2. Trig proof
3. Polynomial factor and solve
4. Transformation graph question
5. Transformation graph question
6. Log equation p.183. p.194 –
7. Ferris wheel type question p.246 ex.4
8. Binomial expansion

✓pathway question (p.339)
✗trig.... point (m,n) determine 3csc θ (p.221 #5)
✗trig.... y= 3cosbx → given the graph, what is b? (p.243 #5)
✗ identifying log function to graph (p.161 #5, p.169 #6,7)
✗ solve a trig. eqn. (p.285)
asymptote question i.e. of y= 2csc 3x
✗ determine an expression equivalent to (double angle ID) p.300 #1 a-j
✗ solve sinx= cosx p.233 #8 #9
✗ tricky log question $\log_x - = -$ determine $\log_x \sqrt{\ }$ p 184
✗ characteristics of horiz. asymptotes (Ch.3 p.138) questions p.134-36