

$$2 \text{a}) \quad \frac{3}{2\sin x} \cdot \left( \frac{\sin x}{\sin x} \right) - \frac{4}{\sin^2 x} \cdot \left( \frac{2}{2} \right) = \frac{3\sin x}{2\sin^2 x} - \frac{8}{2\sin^2 x}$$

$$= \boxed{\frac{3\sin x - 8}{2\sin^2 x}}$$

$$\text{c}) \quad \frac{\tan x}{\tan x} + \frac{1}{\tan x} = \frac{\tan x + 1}{\tan x} \times \frac{\tan^2 x}{1}$$

$$= \boxed{(\tan x + 1) \tan x}$$

$$\text{e}) \quad \sin x \left( \frac{\sin x}{\sin x} \right) + \frac{\cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x}$$

$$= \frac{1}{\sin x} \quad (\text{because } \sin^2 x + \cos^2 x = 1) = \boxed{\csc x}$$

$$\text{g}) \quad \frac{\cos x}{1+\sin x} \left( \frac{\cos x}{\cos x} \right) + \frac{1+\sin x}{\cos x} \left( \frac{1+\sin x}{1+\sin x} \right)$$

$$= \frac{\cos^2 x}{(1+\sin x) \cos x} + \frac{1+\sin x + \sin x + \sin^2 x}{(1+\sin x) \cos x}$$

$$= \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x) \cos x} \quad \text{but } \sin^2 x + \cos^2 x = 1$$

$$= \frac{1 + 1 + 2\sin x}{(1+\sin x) \cos x} = \frac{2 + 2\sin x}{(1+\sin x) \cos x} = \frac{2(1+\sin x)}{(1+\sin x) \cos x}$$

$$= \frac{2}{\cos x} = 2 \cdot \frac{1}{\cos x} = \boxed{2 \sec x}$$

## 6.1 cont'd

3 a)  $1 - \sin^2 x = \boxed{\cos^2 x}$  because  $\sin^2 x + \cos^2 x = 1$

c)  $\tan^2 x - \tan^2 x \sin^2 x = \tan^2 x (1 - \sin^2 x)$

$$= \tan^2 x (\cos^2 x) \quad \text{same reason as a)}$$

$$= \frac{\sin^2 x}{\cos^2 x} (\cos^2 x) \quad \text{because } \frac{\sin x}{\cos x} = \tan x$$

$$= \boxed{\sin^2 x}$$

e)  $\sin^2 x \sec^2 x - \sin^2 x = \sin^2 x (\sec^2 x - 1)$

$$= \boxed{\sin^2 x \tan^2 x} \quad \text{because } 1 + \tan^2 x = \sec^2 x$$

g)  $\cot^4 x + 2\cot^2 x + 1 = (\cot^2 x + 1)(\cot^2 x + 1)$

$$= (\csc^2 x)(\csc^2 x) \quad \text{because } 1 + \cot^2 x = \csc^2 x$$

$$= \boxed{\csc^4 x}$$

i)  $\sin^4 x - \cos^4 x = (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$

$$= 1 (\sin^2 x - \cos^2 x) = \sin^2 x - \cos^2 x -$$

$$= \boxed{(\sin x + \cos x)(\sin x - \cos x)}$$

6.1 cont'd

4. a)  $(\sin x + \cos x)^2 = (\sin x + \cos x)(\sin x + \cos x)$

$$= \sin^2 x + 2 \sin x \cos x + \cos^2 x$$

$$= \boxed{1 + 2 \sin x \cos x}$$

because  $\sin^2 x + \cos^2 x = 1$

c)  $(\csc x - 1)(\csc x + 1) = \csc^2 x + \csc x - \csc x - 1$

$$= \csc^2 x - 1 = \boxed{\cot^2 x} \quad \text{because } 1 + \cot^2 x = \csc^2 x$$

e)  $(\csc x - \cot x)(\csc x + \cot x) = \csc^2 x + \csc x \cot x - \csc x \cot x + \cot^2 x$

$$= \csc^2 x - \cot^2 x = \boxed{1}$$

5 a)  $\sin^2 x - \cos^2 x = \sin^2 x - (1 - \sin^2 x)$

because  
 $\sin^2 x + \cos^2 x = 1$

$$= \sin^2 x - 1 + \sin^2 x$$

$$= \boxed{2 \sin^2 x - 1}$$

d)  $\frac{\sin x + \tan x}{1 + \sec x} = \frac{\sin x + \frac{\sin x}{\cos x}}{1 + \frac{1}{\cos x}} = \frac{\sin x \cdot \frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{1}{\cos x}}$

$$= \frac{\sin x \cos x + \sin x}{\cos x + 1}$$

$$= \frac{\sin x (\cos x + 1)}{\cos x} \times \frac{\cos x}{\cos x + 1}$$

$$= \boxed{\sin x}$$

6.1 cont'd

6 a)  $\sin^2 x - \cos^2 x = 1 - \cos^2 x - \cos^2 x$  because  $\sin^2 x + \cos^2 x = 1$   
=  $1 - 2\cos^2 x$

d)  $\frac{\cot x + \csc x}{\sin x} = \frac{\frac{\cos x}{\sin x} + \frac{1}{\sin x}}{\sin x} = \frac{\cos x + 1}{\sin x}$   
=  $\frac{\cos x + 1}{\sin x} \times \frac{1}{\sin x} = \frac{\cos x + 1}{\sin^2 x} = \frac{\cos x + 1}{1 - \cos^2 x}$   
=  $\frac{\cos x + 1}{(1 + \cos x)(1 - \cos x)} = \frac{1}{1 - \cos x}$

7 a)  $\csc x + \cot x = \frac{1}{\sin x} + \frac{\cos x}{\sin x} = \frac{1 + \cos x}{\sin x}$

d)  $\sec x - \frac{\cos x}{1 + \sin x} = \frac{1}{\cos x} - \frac{\cos x}{1 + \sin x}$   
=  $\frac{1}{\cos x} \cdot \frac{1 + \sin x}{1 + \sin x} - \frac{\cos x}{1 + \sin x} \cdot \frac{\cos x}{\cos x}$   
=  $\frac{1 + \sin x - \cos^2 x}{(\cos x)(1 + \sin x)} = \frac{1 + \sin x - (1 - \sin^2 x)}{\cos x (1 + \sin x)}$   
=  $\frac{1 + \sin x - 1 + \sin^2 x}{(\cos x)(1 + \sin x)} = \frac{\sin x + \sin^2 x}{\cos x (1 + \sin x)} = \frac{\sin x (1 + \sin x)}{\cos x (1 + \sin x)}$   
=  $\frac{\sin x}{\cos x}$

## 6.1 cont'd

$$8 \text{ a) } \frac{\cot x}{1+\sin x} = \frac{\frac{\cos x}{\sin x}}{1+\sin x}$$

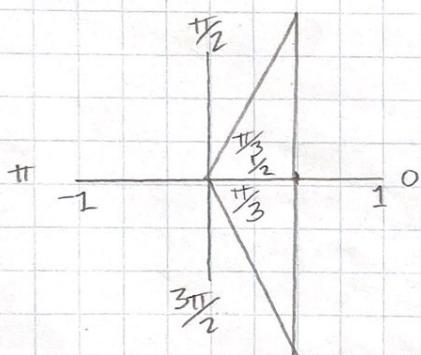
$\sin x \neq 0$   
 $\therefore x \neq 0, \pi$   
 $1+\sin x \neq 0$   
 $\sin x \neq -1$   
 $\therefore x \neq \frac{3\pi}{2}$

$$\text{c) } \frac{1}{2\cos^2 x + \cos x - 1} = \frac{1}{(2\cos x - 1)(\cos x + 1)}$$

$$2\cos x - 1 \neq 0 \quad \text{or} \quad \cos x + 1 \neq 0$$

$$2\cos x \neq 1$$

$$\cos x = -1$$



$$\cos x \neq \frac{1}{2}$$

$$\therefore x \neq \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\therefore x \neq \pi$$

$$9 \text{ a) } (\sec x \cdot \csc x - \cot x)(\sin x - \csc x)$$

$$= \left( \frac{1}{\cos x} \cdot \frac{1}{\sin x} - \frac{\cos x}{\sin x} \right) \left( \sin x - \frac{1}{\sin x} \right)$$

$$= \left( \frac{1}{\cos x \sin x} - \frac{\cos x}{\sin x} \right) \left( \sin x \cdot \frac{\sin x}{\sin x} - \frac{1}{\sin x} \right)$$

$$= \left( \frac{1}{\cos x \sin x} - \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x} \right) \left( \frac{\sin^2 x - 1}{\sin x} \right)$$

$$= \left( \frac{1 - \cos^2 x}{\cos x \sin x} \right) \left( \frac{\sin^2 x - 1}{\sin x} \right) = \left( \frac{\sin^2 x}{\cos x \sin x} \right) \left( \frac{-\cos^2 x}{\sin x} \right)$$

$$= \left( \frac{\sin x}{\cos x} \right) \left( \frac{-\cos^2 x}{\sin x} \right)$$

$$= -\cos x$$

6.1 cont'd

$$9c) \frac{\tan^2 x}{\cos^2 x + \sin^2 x + \tan^2 x} = \frac{\tan^2 x}{1 + \tan^2 x} = \frac{\tan^2 x}{\sec^2 x}$$
$$= \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} = \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1} = \boxed{\sin^2 x}$$

$$g) \frac{\cot x (\sin x + \tan x)}{\csc x + \cot x} = \frac{\frac{\cos x}{\sin x} \left( \sin x + \frac{\sin x}{\cos x} \right)}{\frac{1}{\sin x} + \frac{\cos x}{\sin x}} = \frac{\frac{\cos x + 1}{\sin x}}{\frac{1 + \cos x}{\sin x}}$$
$$= \frac{\cos x + 1}{1 + \cos x} \cdot \boxed{\sin x}$$

$$j) \frac{\csc^2 x + \sec^2 x}{\csc x \sec x} = \frac{\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}}{\frac{1}{\sin x} \cdot \frac{1}{\cos x}} = \frac{\frac{1}{\sin^2 x} \cdot \frac{\cos^2 x}{\cos^2 x} + \frac{1}{\cos^2 x} \cdot \frac{\sin^2 x}{\sin^2 x}}{\frac{1}{\sin x \cos x}}$$
$$= \frac{\frac{\cos^2 x}{\sin^2 x \cos^2 x} + \frac{\sin^2 x}{\sin^2 x \cos^2 x}}{\frac{1}{\sin x \cos x}} = \frac{\frac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x}}{\frac{1}{\sin x \cos x}} \cdot \boxed{\frac{\sin x \cos x}{1}}$$
$$= \frac{1}{\sin x \cos x} = \boxed{\csc x \sec x}$$