Pre-Calc. 12 Ch.1 Practice Test Solutions

*Multiple Choice*

1. **A** is the only sequence that has a common ratio, therefore it is *geometric*. The other three sequences have common differences. They are *arithmetic*.

2. $t_9 = 104 \longrightarrow 104 = a + (9-1)d \longrightarrow 104 = a + 8d$ ①

   $t_{19} = 224 \longrightarrow 224 = a + (19-1)d \longrightarrow \underline{224 = a + 18d}$ ②

   subtract equation ② from ① $\longrightarrow -120 = -10d \longrightarrow d = 12$

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substitute d = 12 into equation ① $\longrightarrow 104 = a + (8)(12)$

$104 = a + 96$

$104 - 96 = a \longrightarrow a = 8$

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Therefore, $t_{29} = 8 + (29-1)(12) = 8 + (28)(12) = 8 + 336 =$ **344 A**

3. This is a question where you have to investigate by checking each sequence. In this case we can rule out choices **A** and **D** because each term in both sequences ends with 5.
   Let's check choice **C** where a = 3 and r = 2 and $t_n = 6144$
   Using $t_n = ar^{n-1} \longrightarrow 6144 = (3)2^{n-1}$

   $2048 = 2^{n-1}$ but, $2048 = 2^{11}$

   $\therefore 2^{11} = 2^{n-1}$

   $\therefore 11 = n-1$ This means that 6144 is a term in sequence **C**

*Short Answer*

1. $d = t_{n+1} - t_n = -19 - (-20) = -19 + 20 =$ **1**

2. Use $t_n = a + (n-1)d$ , where a = -6, n = 15 and d = 5

   $\therefore t_{15} = -6 + (15-1)(5)$
   $= -6 + (14)(5)$
   $= -6 + 70$
   $=$ **64**

3. $t_1 =$ **16** $t_2 = 16 + (-3) =$ **13** $t_3 = 13 + (-3) =$ **10** $t_4 = 10 + (-3) =$ **7**

4. $t_5 = 14.4 \longrightarrow$ $14.4 = a + (5-1)d \longrightarrow$ $14.4 = a + 4d$ ①

$t_{12} = 29.1 \longrightarrow$ $29.1 = a + (12-1)d \longrightarrow$ $\underline{29.1 = a + 11d}$ ②

subtract equation ② from ① $\longrightarrow$ $-14.7 = -7d \longrightarrow d = 2.1$

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substitute $d = 2.1$ into equation ① $\longrightarrow$ $14.4 = a + (4)(2.1)$

$14.4 = a + 8.4$

$14.4 - 8.4 = a \longrightarrow a = 6$

$\therefore\quad t_{21} = 6 + (21-1)(2.1)$

$= 6 + (20)(2.1)$

$= 6 + 42 =$ **48**

5. Use $t_n = a + (n-1)d$ where $t_n = 306$, $a = -18$, and $d = 9$

$\therefore\quad 306 = -18 + (n-1)9$

$306 = -18 + 9n - 9$

$306 = -27 + 9n$

$306 + 27 = 9n$

$333 = 9n \longrightarrow$ **n = 37**

6. Use $S_n = \frac{n}{2}(2a + (n-1)d)$ where $a = -4$, $n = 13$, and $d = 1.5$

$\therefore\quad S_{13} = \frac{13}{2}(2(-4) + (13-1)1.5$

$= 6.5\,(-8 + (12)1.5)$

$= 6.5\,(-8 + 18)$

$= 6.5\,(10) =$ **65**

7. We need to determine the common difference first *before* using the equation $S_n = \frac{n}{2}(2a + (n-1)d)$

STEP 1: Use $t_n = a + (n-1)d$ to find d, where $t_6 = -10$, $a = -5$, and $n = 6$

$\therefore\quad -10 = -5 + (6 - 1)d$

$-10 = -5 + 5d$

$-10 + 5 = 5d$

$-5 = 5d \longrightarrow d = -1$

STEP 2: Now use $S_n = \frac{n}{2}(2a + (n-1)d)$ where n = 15, a = -5, and d = -1

$$\therefore\ S_{15} = \frac{15}{2}(2(-5) + (15-1)(-1))$$

$$= 7.5\ (-10\ -14\ ) = 7.5\ (\ -24\ ) = \underline{\mathbf{-180}}$$

8. Use $S_n = \frac{n}{2}(2a + (n-1)d)$ where, n = 22, a = 4, and d = 5.5

$$\therefore\ S_{22} = \frac{22}{2}(2(4) + (22-1)(5.5))$$
$$= 11\ (8 + (21)(5.5))$$
$$= 11\ (8 + 115.5) = 11\ (123.5) = \underline{\mathbf{1358.5}}$$

9. Use $t_n = ar^{n-1}$ where, $t_1 = a$, $t_6 = -2430$, r = 3, and n = 6

$$\therefore\ -2430 = a{\cdot}3^{6-1}$$

$$-2430 = a{\cdot}3^5$$

$$-2430 = 243a \longrightarrow \underline{\mathbf{a = -10}}$$

10. In this case we can see $\mathbf{r} = \frac{40}{80} = \frac{\mathbf{1}}{\mathbf{2}} = \underline{\mathbf{0.5}}$, a = 80 and n = 7

$$\therefore\ t_7 = 80\ (0.5)^{7-1}$$
$$t_7 = 80\ (0.5)^6$$
$$t_7 = 80\ (0.15625) = \underline{\mathbf{1.25}}$$

11. In order to determine the number of terms in the sequence, we need to know the ratio

$t_4 = 3{\cdot}r^{4-1} \longrightarrow -24 = 3{\cdot}r^3 \longrightarrow -8 = r^3 \longrightarrow r = -2$

if $t_n = -6144$ then $-6144 = 3(-2)^{n-1}$

$-2048 = (-2)^{n-1}$ but $-2048 = (-2)^{11}$

$(-2)^{11} = (-2)^{n-1} \longrightarrow 11 = n - 1 \longrightarrow \underline{\mathbf{n = 12}}$

12. $t_1 = \mathbf{2500}$, $t_2 = 2500\ (\frac{-1}{5}) = \mathbf{-500}$, $t_3 = -500\ (\frac{-1}{5}) = \mathbf{100}$, $t_2 = 100\ (\frac{-1}{5}) = \mathbf{-20}$

13. Use the equation $S_n = \frac{a(1-r^n)}{1-r} = \frac{-2(1-2^{15})}{1-2} = \underline{\mathbf{-65534}}$

14. Use $S_n = \frac{a(1-r^n)}{1-r}$, $\therefore$ $44286 = \frac{a(1-(-3)^{10})}{1-(-3)} \longrightarrow 44286 = \frac{a(-59048)}{4} \longrightarrow 4(44286) = -59048a$

$$a = \frac{4(44286)}{-59048} = \underline{\mathbf{-3}}$$

15. $r = \frac{256}{1024} = \frac{1}{4}$ Use $S_n = \frac{a-rl}{1-r}$ where, a = 1024, r = .25 and l = 4

$\therefore\ S_n = \frac{1024-.25(4)}{1-.25} = \frac{1023}{.75} = \underline{\mathbf{1364}}$

16. Use $S = \frac{a}{1-r}$ where, a = 27 and $r = \frac{9}{27} = \frac{1}{3}$

$\therefore\ S = \frac{27}{1-\frac{1}{3}} = \frac{27}{\frac{2}{3}} = \underline{\mathbf{40.5}}$

17. The ratio of this series is $\frac{5}{2}$ $\therefore$ **this series does not have a sum because | r | > 1**

18. Use $S = \frac{a}{1-r}$ where a = -5 and S = - 15 $\therefore$ $-15 = \frac{-5}{1-r} \longrightarrow -15(1-r) = -5$

$-15 + 15r = -5$

$15r = -5 + 15$

$15r = 10 \longrightarrow \mathbf{r = \frac{2}{3}}$

19. $t_1 = \mathbf{-2}$, $t_2 = -2\left(\frac{2}{3}\right) = \mathbf{\frac{-4}{3}}$, $t_3 = \left(\frac{-4}{3}\right)\left(\frac{2}{3}\right) = \mathbf{\frac{-8}{9}}$, $t_4 = \left(\frac{-8}{9}\right)\left(\frac{2}{3}\right) = \mathbf{\frac{-16}{27}}$

20. Use $t_n = a + (n-1)d$ where: a = 15, d = 5, and n = 8

$\therefore\ t_8 = 15 + (8-1)(5) = 15 + 7(5) = 15 + 35 = \underline{\mathbf{50}}$

21. $t_9 = 17.8 \longrightarrow 17.8 = a + (9-1)d \longrightarrow 17.8 = a + 8d$ ①

$t_{14} = 15.8 \longrightarrow 15.8 = a + (14-1)d \longrightarrow \underline{15.8 = a + 13d}$ ②

subtract equation ② from ① $\longrightarrow$ $2 = -5d \longrightarrow d = \frac{2}{-5} = -0.4$

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substitute d = – 0.4 into equation ① $\longrightarrow$ $17.8 = a + (8)(-0.4)$

$17.8 = a - 3.2$

$17.8 + 3.2 = a \longrightarrow a = 21$

$\therefore$ $11.4 = 21 + (n-1)(-0.4) \longrightarrow 11.4 = 21 - 0.4n + 0.4 \longrightarrow 11.4 = 21.4 - 0.4n$

$11.4 - 21.4 = -0.4n$

$-10 = -0.4n$

$\underline{\mathbf{n = 25}}$

22. Use $S_n = \frac{n}{2}(2a + (n-1)d)$ where: $a = -5$, $d = -3$, and $n = 9$

$\therefore$ $S_n = \frac{9}{2}(2(-5) + (9-1)(-3)) = 4.5(-10 + 8(-3)) = 4.5(-10 - 24) = 4.5(-34) =$ **$-153$**

23. Determine a, the first term, by using $S_n = \frac{n}{2}(a + l)$ where: $S_{20} = 350$, $n = 20$, $l = t_{20} = 27$

$\therefore$ $350 = \frac{20}{2}(a + 27) \longrightarrow 350 = 10(a + 27) \longrightarrow 350 = 10a + 270$

$350 - 270 = 10a$

$80 = 10a \longrightarrow a = 8$

Since the difference, d = 1, the first 3 terms of the series are **8 + 9 + 10**

24. Determine **a** by using $S_n = \frac{n}{2}(a + l)$ where: $l = t_{13} = 101$, $S_{13} = 689$, and $n = 13$

$\therefore$ $689 = \frac{13}{2}(a + 101) \longrightarrow 689 = 6.5(a + 101) \longrightarrow 689 = 6.5a + 656.5$

$689 - 656.5 = 6.5a$

$32.5 = 6.5a \longrightarrow$ **$a = 5$**

25. $S_{26} - S_{25} = t_{26} \longrightarrow -3328 - (-3075) = t_{26} \longrightarrow t_{26} = -253$

Using $t_n = a + (n-1)d$ we can see that $-253 = a + (26-1)(-10)$

$-253 = a + 25(-10)$

$-253 = a - 250$

$-253 + 250 = a \longrightarrow$ **$a = -3$**

26. $S_{20} - S_{19} = t_{20} \longrightarrow -1760 - (-1577) = t_{20} \longrightarrow t_{20} = -183$

Using $t_n = a + (n-1)d$ we can see that $-183 = a + (20-1)(-10)$

$-183 = a + 19(-10)$

$-183 = a - 190$

$-183 + 190 = a \longrightarrow a = 7$

The first 4 terms of the series are **7, –3, –13, –23**

27. Use $t_n = ar^{n-1}$ where: a = \$1450, r = 1.13 , and n = 49 – 20 = 29 years

$\therefore$ $t_n = \$1450(1.13)^{9-1} = \$1450(1.13)^8 =$ **\$50 192.97**

28. $r = \frac{15}{75} = 0.2$ Use the equation $S_n = \frac{a(1-r^n)}{1-r} = \frac{75(1-0.2^5)}{1-0.2} =$ **93.72**

29. Given: $S_4 = 21$, $S_5 = 45$, $S_6 = 93$

This means $S_5 - S_4 = t_5$ and $S_6 - S_5 = t_6$

$45 - 21 = t_5$ $\quad$ $93 - 45 = t_6$

$t_5 = 24$ $\quad$ $t_6 = 48$

∴ the common ratio is $48 \div 24 =$ **2**

30. Use the equation $S = \frac{a}{1-r}$ where: $a = -5$ and $r = \frac{1}{4}$

$$\therefore S = \frac{-5}{1-\frac{1}{4}} = \frac{-5}{\frac{3}{4}} = -5 \times \frac{4}{3} = \frac{-20}{3}$$

31. In order for a series to have a sum, the $|r|$ must be $< 1$. The ratio of this series is $\frac{1}{2}$ ∴ it has a sum.

$$S = \frac{a}{1-r} = \frac{2}{\frac{1}{2}} = 2 \times \frac{2}{1} = \mathbf{4}$$

*Problem*

1. The cost for one ticket in section 1 is \$95 = a
   The cost for one ticket in section 2 is \$90
   The cost for one ticket in section 3 is \$85 ∴ The common difference is – 5
   The number of tickets, n = 10

   Using $S_n = \frac{n}{2}(2a + (n-1)d$ we get $S_{10} = \frac{10}{2}(2(95) + (10-1)(-5))$
   $= 5(190 + 9(-5))$
   $= 5(190 - 45) = 5(145) =$ **\$725**

2.

| **CAR** | **TRUCK** |
|---|---|
| $a = \$44500$<br>$r = 1 - 0.15 = 0.845$<br>$n = 10$ (not $(n-1)$ years because n is the number of years passed)<br><br>$t_n = ar^n = 44500\,(0.845)^{10} = \$8258.99$ | $a = \$25000$<br>$r = 1 - 0.075 = 0.925$<br>$n = 10$<br><br><br>$t_n = ar^n = 25000\,(0.925)^{10} = \$11\,464.56$ |

**The Truck is worth more.**

3. Use $S_n = \frac{a(1-r^n)}{1-r}$ where: $S_n = -635$, $r = 2$, and $n = 7$

$$-635 = \frac{a(1-2^7)}{1-2} \longrightarrow -635 = \frac{a(-127)}{-1} \longrightarrow 635 = -127a \longrightarrow \mathbf{\underline{a = -5}}$$

4. a) If $t_n = -5(\frac{-1}{9})^{n-1}$

$$t_1 = -5(\tfrac{-1}{9})^{1-1} = -5\,(\tfrac{-1}{9})^0 = -5\,(1) = \mathbf{-5}$$

$$t_2 = -5(\tfrac{-1}{9})^{2-1} = -5\,(\tfrac{-1}{9})^1 = -5\,(\tfrac{-1}{9})) = \mathbf{\frac{5}{9}}$$

$$t_3 = -5(\tfrac{-1}{9})^{3-1} = -5\,(\tfrac{-1}{9})^2 = -5\,(\tfrac{1}{81})) = \mathbf{\frac{-5}{81}}$$

$$t_4 = -5(\tfrac{-1}{9})^{4-1} = -5\,(\tfrac{-1}{9})^3 = -5\,(\tfrac{-1}{729})) = \mathbf{\frac{5}{729}}$$

b) The common ratio of this series is $r = \frac{-1}{9}$ , Since $|\,r\,| < 1$ this series **<u>CONVERGES</u>**

c) $S = \frac{-5}{1-\frac{-1}{9}} = \frac{-5}{\frac{10}{9}} = -5 \times \frac{9}{10} = \mathbf{\underline{-4.5}}$

5. The first three terms of this series are 2(5) + 3 = 13
2(6) +3 = 15
2(7) + 3 = 17
We can see this is an arithmetic series, where a = 13, d = 2, and n = 25 – 5 + 1 = 21 terms

$\therefore$ use $S_{21} = \frac{21}{2}(2(13) + (21-1)(2) = 10.5\,(26 + (20)(2)) = 10.5\,(26 + 40) = 10.5\,(66) =$ **<u>693</u>**

6. Total vertical distance travelled is $2 \times \frac{10}{1-0.75} - 10\text{ m} = 2 \times \frac{10}{.25} - 10\text{ m} = 2 \times 40 - 10\text{ m} =$ **<u>70 m</u>**

7. Use $S_n = \frac{a(1-r^n)}{1-r}$ where: $S_{12} = 6265.65$, $r = \frac{7}{5}$, and $n = 12$

$$S_{12} = \frac{a(1-\frac{7}{5}^{12})}{1-\frac{7}{5}} \longrightarrow 6265.65 = \frac{a(1-\frac{7}{5}^{12})}{-\frac{2}{5}} \longrightarrow \frac{6265.65(-\frac{2}{5})}{(1-\frac{7}{5}^{12})} = a$$

**<u>a = 45.00 cm</u>**