

PRACTICE EXERCISES

1. The graph of $y = (3x)^3$ is the graph of $y = x^3$ stretched horizontally about the y -axis by a factor of $\frac{1}{3}$.

2. The graph of $y = 2x^2 + 2$ is the graph of $y = x^2 + 1$ stretched vertically about the x -axis by a factor of 2.

3. Given $y = x^2 + 3x$, write an equation that would

a) stretch the graph of this function horizontally about the y -axis by a factor of $\frac{1}{2}$
replace x with $2x$

$$y = (2x)^2 + 3(2x) \longrightarrow y = 4x^2 + 6x$$

b) stretch the graph of this function vertically about the x -axis by a factor of $\frac{1}{2}$
replace y with $2y$

$$2y = x^2 + 3x \longrightarrow y = \frac{1}{2}(x^2 + 3x)$$

4. The graph of $y = f(x)$ contains the point $(6, 3)$. What corresponding point is on the graph of

a) $y = f(3x)$?
horiz. comp. factor $\frac{1}{3}$
 $(2, 3)$

b) $y = -2f(x)$?
① replace y with $-y$ ② vert. exp. factor 2
 $(6, -6)$

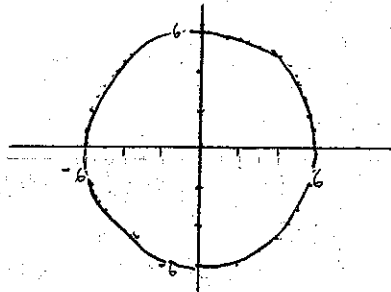
5. For $y = |x + 3|$, write an equation that represents the graph of this function stretched horizontally about the y -axis by a factor of 2 and that reflects it in the y -axis.

$$y = |x + 3| \xrightarrow{\substack{\downarrow \\ \text{replace } x \text{ with } \frac{1}{2}x}} y = \left| \frac{1}{2}x + 3 \right| \xrightarrow{\substack{\downarrow \\ \text{replace } x \text{ with } -x}} y = \left| -\frac{1}{2}x + 3 \right|$$

6. The graph of $y = 5\sqrt{x}$ is the graph of $y = \sqrt{x}$, stretched vertically about the x -axis by a factor of 5.

7. The graph of $y = \left(\frac{1}{2}x\right)^3$ is the graph of $y = (x)^3$ stretched horizontally about the y -axis by a factor of 2.

8. The graph of $x^2 + y^2 = 9$ is a circle with centre $(0, 0)$ and radius 3. Sketch $\left(\frac{1}{2}x\right)^2 + \left(\frac{1}{2}y\right)^2 = 9$, showing the x - and y -intercepts. Give the domain and range of this new relation.



$$D: -6 \leq x \leq 6$$

$$R: -6 \leq y \leq 6$$

$$x\text{-int @ } (-6, 0) \quad (6, 0)$$

$$y\text{-int @ } (0, -6) \quad (0, 6)$$

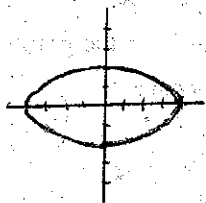
9. In general, $ky = f(x)$ is the graph of $y = f(x)$ stretched vertically about the x -axis by a factor of $\frac{1}{k}$.
10. For $y = \sqrt{x}$, write the equation that represents the graph of this function stretched horizontally about the y -axis by a factor of 2 and vertically about the x -axis by a factor of 3.

$$y = \sqrt{\frac{1}{2}x}$$

$$\frac{1}{3}y = \sqrt{x}$$

$$y = 3\sqrt{\frac{1}{2}x}$$

11. The graph of $y = 8x^3$ could be the graph of $y = x^3$, if it is stretched
- a) vertically about the x -axis by a factor of 8
- b) horizontally about the y -axis by a factor of $\frac{1}{2}$ $\xrightarrow{\text{THINK...}}$ $y = (2x)^3$
12. If the graph of $y = f(x)$ contains the point (a, b) , the graph of $y = 2f(3x)$ contains the corresponding point $(\frac{a}{3}, 2b)$.
13. The graph of $x^2 + y^2 = 16$ is a circle with centre $(0, 0)$ and radius 4. Sketch the graph of $x^2 + (2y)^2 = 16$, showing the x - and y -intercepts. Give the domain and range of this new relation.



vert. comp factor $\frac{1}{2}$
 x -int $(-4, 0) (4, 0)$
 y -int $(0, 2) (0, 2)$

14. The graph of $y = f(x)$ has a point (a, b) . What corresponding point is on the graph of $y = -2f(x)$?

$$(a, -2b)$$

15. If the graph of $y = \sqrt{x}$ is stretched horizontally about the y -axis by a factor of 4 and reflect in the y -axis, then what is the equation of the transformed graph?

$$y = \sqrt{-\frac{1}{4}x}$$

16. If the graph of $y = |x| - 2$ is stretched vertically about the x -axis by a factor of 3 and reflect in the x -axis, then what is equation of the transformed graph?

replace y with $\frac{y}{3}$ and y with $-y$

$$\therefore -\frac{y}{3} = |x| - 2 \longrightarrow y = -3(|x| - 2) \longrightarrow y = -3|x| + 6$$

17. If the graph of $y = x^3 + 3$ is stretched horizontally about the y -axis by a factor of $\frac{1}{2}$, stretched vertically about the x -axis by a factor of 2, and reflect in the x -axis, then what is equation of the transformed graph?

$$y = (2x)^3 + 3$$

$$\frac{1}{2}y = (2x)^3 + 3$$

$$-\frac{1}{2}y = (2x)^3 + 3 \longrightarrow y = -2(2x)^3 - 6$$

18. If the graph of $y = \frac{1}{x-2}$ is stretched vertically about the x -axis by a factor of $\frac{1}{3}$, stretched horizontally stretch about the y -axis by a factor of 2, and reflect in the y -axis, then what is equation of the transformed graph?

$$3y = \frac{1}{x-2} \longrightarrow 3y = \frac{1}{\frac{x}{2}-2} \longrightarrow 3y = \frac{1}{-\frac{x}{2}-2} \longrightarrow y = \frac{2}{-3(x+4)}$$

For each of questions 19 to 24, describe how the graph of the first function can be transformed to give the graph of the second function.

19. $y = |x|$, $y = -2|x|$

vert. exp. factor 2
reflection in x -axis

20. $y = x^2$, $y = -\left(\frac{1}{3}x\right)^2$

horiz. exp. factor 3
reflection in x -axis

21. $y = \frac{2}{x+2}$, $4y = \frac{-2}{x+2}$

vert. comp. factor $\frac{1}{4}$
ref. in x -axis

22. $y = x^3 - x$, $-2y = (3x)^3 - (3x)$

horiz. comp. fact. $\frac{1}{3}$
ref. in x -axis
vert. comp. fact. $\frac{1}{2}$

23. $y = 3^x$, $y = \frac{2}{3^x} \longrightarrow y = 2 \cdot 3^{-x}$

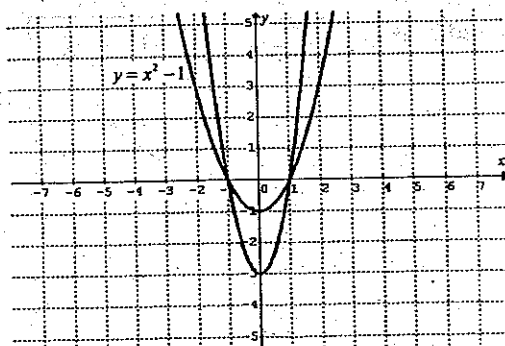
ref. in y -axis
vert. exp. factor 2

24. $y = \frac{1}{x+3}$, $y = \frac{-2}{3x+3}$

vert. exp. factor 2
horiz. comp. factor $\frac{1}{3}$
ref. in x -axis

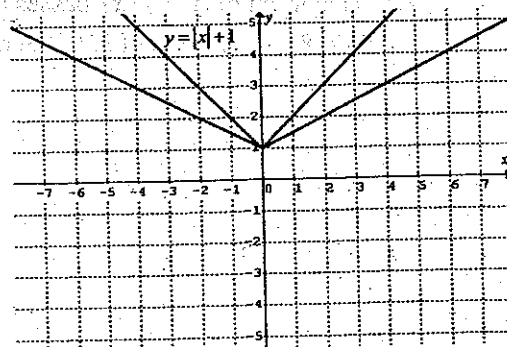
For each of questions 25 to 28, the graph that is not labelled is obtained by a stretch of the graph of the function that is labelled. Write the equation for each of the functions represented by the graphs that are not labelled.

25.



$$\frac{1}{3}y = x^2 - 1$$

26.



$$y = \left|\frac{1}{2}x\right| + 1$$