Pre-Calculus 12
Ch.5 Test - Trigonometry

Name: KEY
Score: /28

Practice Test

Multiple Choice
Identify the choice that best completes the statement or answers the question.

1. Convert 3π radians to degrees.

$\pi = 180° \longrightarrow 3\pi = 3 \times 180° = \underline{540°}$

a. 0.16°
b. 9.42°
c. 540°
d. 1080°

2. Convert 420° to radians.

$\frac{\pi}{x} = \frac{180}{420} \longrightarrow \frac{420\pi}{180} = x$

$x = \frac{42\pi}{18} = \boxed{\frac{7\pi}{3}}$

a. $\frac{\pi}{6}$
b. $\frac{\pi}{3}$
c. $\frac{4\pi}{3}$
d. $\frac{7\pi}{3}$

3. Given a circle with radius 10 cm, calculate the length of the arc which contains a sector angle $\theta = 2$ radians.

$a = r\theta$

$a = 10\text{cm} \times 2$

$a = \underline{20\text{cm}}$

a. 5π cm
b. 10π cm
c. 10 cm
d. 20 cm

4. A circle has a radius of 4 cm. If the length of arc AB is 3π cm, determine the measure of the central angle in radians.

$\theta = \frac{a}{r} \longrightarrow \theta = \frac{3\pi \cancel{\text{cm}}}{4 \cancel{\text{cm}}}$

a. $\frac{3\pi}{4}$
b. $\frac{4\pi}{3}$
c. $\frac{3\pi}{2}$
d. 3π

5. A circle has a radius of 20 cm. Determine the length of the arc subtended by a central angle of 135°.

convert 135° to radians

$\frac{\pi}{x} = \frac{180}{135} \longrightarrow \frac{135\pi}{180} = x$ ÷ by 45 ÷ by 45

$x = \frac{3\pi}{4}$

$a = r\theta$

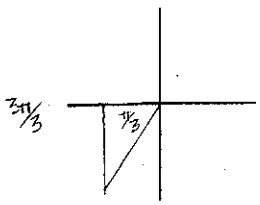
$\therefore a = \cancel{20}^{5}\text{cm} \times \frac{3\pi}{\cancel{4}}$

$a = \underline{15\pi \text{ cm}}$

a. $\frac{3\pi}{4}$ cm
b. $\frac{80}{3\pi}$ cm
c. 5π cm
d. 15π cm

6. What is the reference angle of $\frac{4\pi}{3}$?

a. $\frac{\pi}{6}$
b. $\frac{\pi}{3}$
c. $\frac{2\pi}{3}$
d. $\frac{4\pi}{3}$



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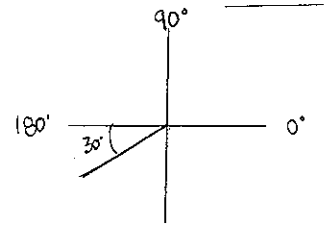
7. Find the reference angle for 210°.

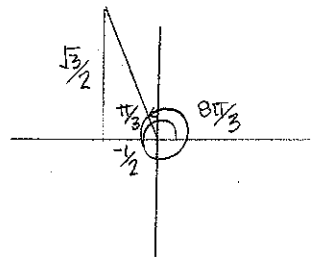
210° is 30° below the horizontal axis

(a.) 30°
b. $\frac{7\pi}{6}$
c. 150°
d. $\frac{5\pi}{6}$



8. Determine the exact value of $\tan\frac{8\pi}{3}$

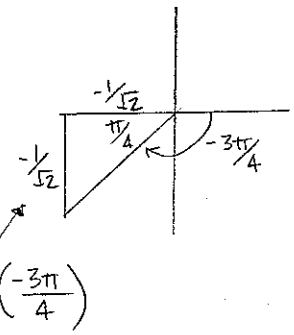
(a.) $-\sqrt{3}$
b. $-\frac{1}{\sqrt{3}}$
c. $\frac{1}{\sqrt{3}}$
d. $\sqrt{3}$



$\therefore \tan\theta = \frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}} = -\sqrt{3}$

9. Determine the exact value of $\sin\left(-\frac{3\pi}{4}\right)$.

a. $-\sqrt{2}$
(b.) $-\frac{1}{\sqrt{2}}$
c. $\frac{1}{\sqrt{2}}$
d. $\sqrt{2}$



10. Solve: $\csc x = 2,\ 0 \le x < 2\pi$

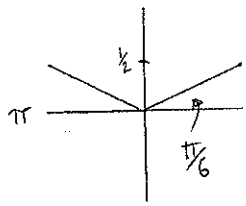
(a.) $x = \frac{\pi}{6}, \frac{5\pi}{6}$
b. $x = \frac{\pi}{6}, \frac{11\pi}{6}$
c. $x = \frac{\pi}{3}, \frac{2\pi}{3}$
d. $x = \frac{\pi}{3}, \frac{4\pi}{3}$

if $\csc x = 2$

then $\frac{1}{\sin x} = 2$

$\therefore\ 1 = 2\sin x$

$\frac{1}{2} = \sin x$



$\boxed{x = \pi/6}$

$x = \pi - \pi/6$

$= \frac{6\pi}{6} - \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$

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$\frac{1}{\cos x}$ ← $\cos x \neq 0$

$\sin x \neq 0$

11. What is (are) the restriction(s) for the expression $\frac{\sec x}{\sin x}$?

a. $\sec x \neq 0$
b. $\sin x \neq 0$
c. $\cos x \neq 0$
d. $\sin x \neq 0,\ \cos x \neq 0$

12. If $\csc x > 0$ and $\cot x < 0$, in which quadrant does the terminal side of position angle x lie?

a. I
b. II
c. III
d. IV

S	A
T	C

- if $\csc x > 0$ then $\sin x > 0$ - quad I and II
- if $\cot x < 0$ then $\tan < 0$ - quad II and IV

13. If $\sin x = \frac{3}{4}$, determine the smallest positive angle x, in radians.

a. 0.68
b. 0.72
c. 0.85
d. 1.47

use calculator in radian mode

if $\sin x = 3/4$

then $x = \sin^{-1}(3/4) = 0.85$

14. For what value of x is $\frac{\sin x}{1+\cos x}$ undefined, where $0 \le x < 2\pi$.

a. 0
b. $\frac{\pi}{2}$
c. π
d. $\frac{3\pi}{2}$

$1 + \cos x \neq 0$

$\cos x \neq -1$

$x = \pi$

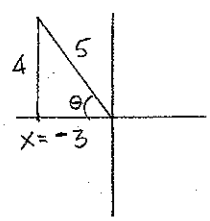
15. Given $\sin\theta = \frac{4}{5}$ and $\sec\theta < 0$, determine the ratio for $\cot\theta$.

quad II or III

quad I or II

∴ we are in quad II

a. $-\frac{4}{3}$
b. $-\frac{3}{4}$
c. $\frac{3}{4}$
d. $\frac{4}{3}$



$x^2 + 4^2 = 5^2$

$x^2 = 5^2 - 4^2$

$x^2 = 9$

$x = -3$

$\cot\theta = \frac{-3}{4}$

16. What is the amplitude of the graph of the function $f(x) = -2\sin x + 5$

a. -2
b. 2
c. 3
d. 7

AMPLITUDE = $|-2|$

17. The graph of the function $f(x) = \sin x$ is translated 4 units up. What is an equation of the new function?

a. $f(x) = \sin x + 4$
b. $f(x) = \sin x - 4$
c. $f(x) = \sin(x+4)$
d. $f(x) = \sin(x-4)$

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18. What is the period of the function $f(x) = 2\sin\frac{1}{2}x$?

a. $\frac{1}{2}$
b. π
c. 2π
d. 4π

period $= \frac{2\pi}{b}$, in this case $b = \frac{1}{2}$

$\therefore$ period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

19. What is the phase shift of the function $f(x) = \sin\left(2x - \frac{\pi}{6}\right) + \frac{\pi}{2}$

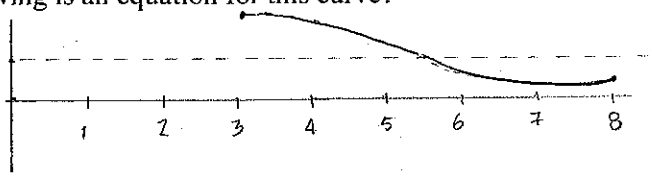
a. $\frac{\pi}{12}$
b. $\frac{\pi}{6}$
c. $\frac{\pi}{3}$
d. $\frac{\pi}{2}$

phase shift is $\frac{\pi/6}{2} = \frac{\pi}{12}$

or// $\sin\left(2\left(x - \frac{\pi}{12}\right)\right)$

phase shift

20. A cosine curve has a maximum point at (3, 20) and the nearest minimum point to the right of this point is (8, 4). Which of the following is an equation for this curve?



a. $y = 8\cos\frac{2\pi}{5}(x+3) + 12$
b. $y = 8\cos\frac{2\pi}{5}(x-3) + 12$
c. $y = 8\cos\frac{\pi}{5}(x+3) + 12$
d. $y = 8\cos\frac{\pi}{5}(x-3) + 12$

period is $2 \times (8-3) = 2 \times 5 = 10$

v.d. $= \frac{M+m}{2} = \frac{20+4}{2} = 12$

$\frac{2\pi}{b} = 10$

$\frac{2\pi}{10} = b$ $\boxed{b = \frac{\pi}{5}}$

phase shift is 3 units RIGHT

so, $(x-3)$

Short Answer (SHOW YOUR WORK)

21. Determine one positive and one negative coterminal angle to –250°

$-250° + 360° = \boxed{110°}$

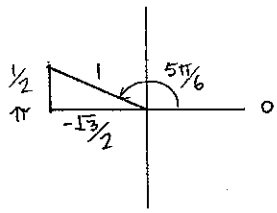
$-250° - 360° = \boxed{-610°}$

22. Determine one positive and one negative coterminal angle to $\frac{7\pi}{15}$

$\frac{7\pi}{15} + 2\pi = \frac{7\pi}{15} + \frac{30\pi}{15} = \boxed{\frac{37\pi}{15}}$

$\frac{7\pi}{15} - 2\pi = \frac{7\pi}{15} - \frac{30\pi}{15} = \boxed{\frac{-23\pi}{15}}$

23. Determine the exact value of $\tan\frac{5\pi}{6}$ without a calculator.



$\tan\frac{5\pi}{6} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \boxed{-\frac{1}{\sqrt{3}}} \times \frac{\sqrt{3}}{\sqrt{3}}$

or// $\boxed{-\frac{\sqrt{3}}{3}}$

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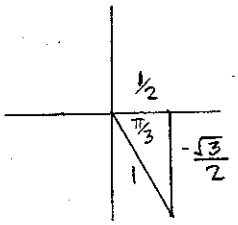
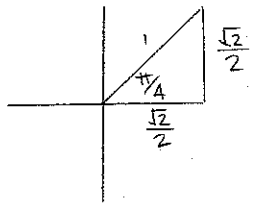
24. Determine the exact value of $\csc \frac{5\pi}{3}$ without a calculator.



$\csc \frac{5\pi}{3} = \frac{\text{hyp.}}{\text{opp.}} = \frac{1}{-\frac{\sqrt{3}}{2}} = \boxed{-\frac{2}{\sqrt{3}}}$

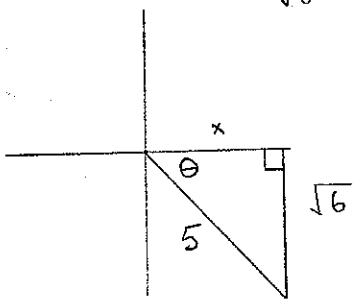
25. Find the exact value of $2\cos^2 \frac{\pi}{4}$



$2\left(\cos\frac{\pi}{4}\right)^2 = 2\left(\frac{\sqrt{2}}{2}\right)^2 = 2\left(\frac{2}{4}\right)$

$= \frac{4}{4} = \boxed{1}$

26. Determine the exact values of the coordinates of the point that is 5 units from the origin in the quadrant IV, if θ is a positive angle and $\csc\theta = -\frac{5}{\sqrt{6}}$. $\longrightarrow \frac{\text{hyp}}{\text{opp}}$



$x^2 + \sqrt{6}^2 = 5^2$

$x^2 + 6 = 25$

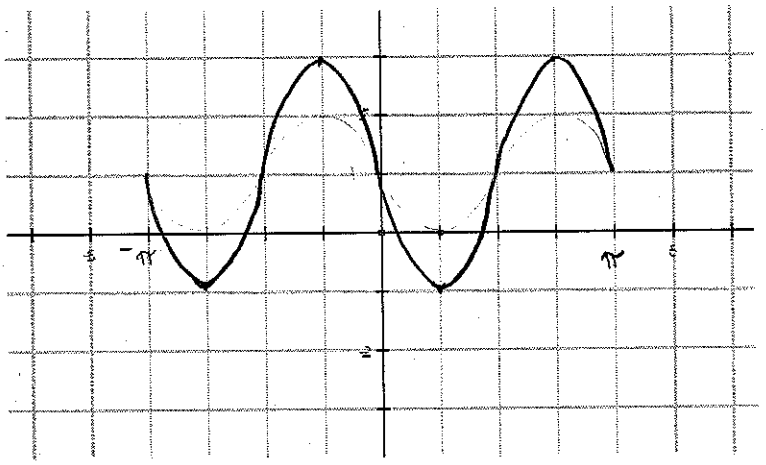
$x^2 = 19$

$x = \sqrt{19}$

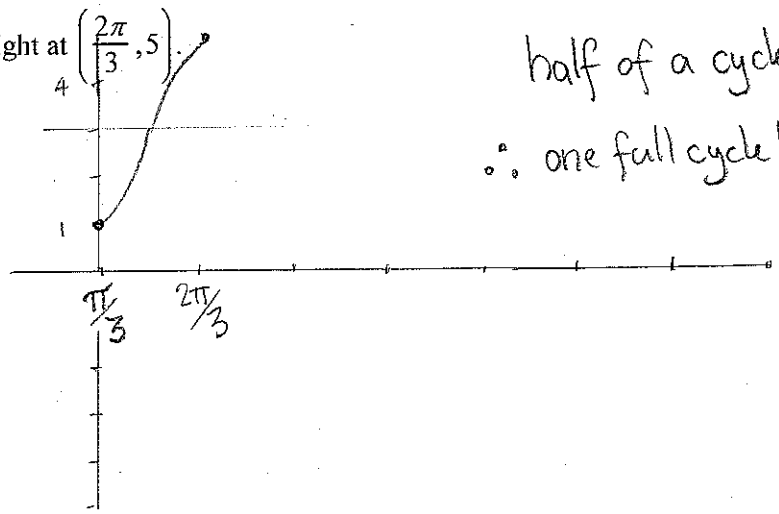
$\therefore$ coordinate of the point is $(\sqrt{19}, -\sqrt{6})$

27. Graph at least two complete periods of the equation y = -2 sin 2x + 1 .

period $= \frac{2\pi}{2} = \pi$



28. Find the period of the sine function which has a minimum point at $\left(\frac{\pi}{3}, 1\right)$ and its nearest maximum point to the right at $\left(\frac{2\pi}{3}, 5\right)$.

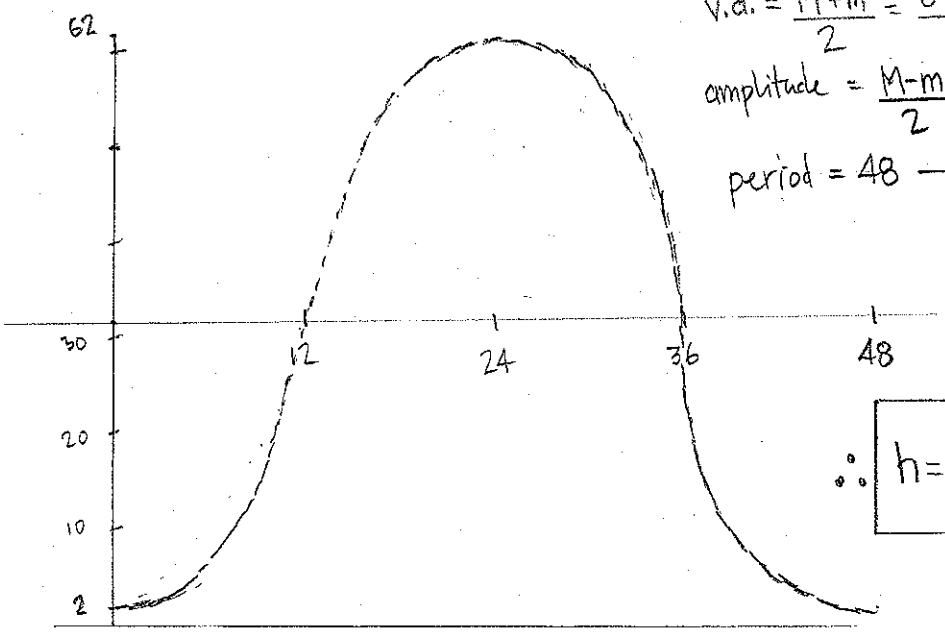


half of a cycle is $\pi/3$ radians

$\therefore$ one full cycle has a period $= \boxed{\frac{2\pi}{3}}$

29. A Ferris wheel with a diameter of 60 m rotates once every 48 seconds. At time t = 0, a rider is at his lowest height which is 2 m above the ground.

Determine a sinusoidal equation that gives the height, h, of the rider above the ground as a function of the elapsed time, t, where he is in metres and t is in seconds.



$\text{v.a.} = \frac{M+m}{2} = \frac{62+2}{2} = \frac{64}{2} = 32$

$\text{amplitude} = \frac{M-m}{2} = \frac{62-2}{2} = \frac{60}{2} = 30$

$\text{period} = 48 \longrightarrow \frac{2\pi}{b} = 48 \longrightarrow \frac{2\pi}{48} = b$

$b = \frac{\pi}{24}$

$\therefore \boxed{h = -30\cos\frac{\pi}{24}t + 32}$