6.6 Chapter Review

Trigonometry (Part I) – Multiple-choice Review Questions

1. Determine the amplitude and period of $y = -2\cos\left(\frac{\pi}{2}x - \pi\right) + 3$.

a) -2 ; 1
b) 2 ; 1
c) -2 ; 4
d) 2 ; 4

period $= 2\pi / \frac{\pi}{2} = 2\pi \times 2/\pi = 4$

amplitude $= 2$

2. Determine the phase shift and vertical displacement of $y = -2\cos\left(\frac{\pi}{2}x - \pi\right) + 3$.

a) $-\pi$; 3
b) π ; -3
c) -2 ; 3
d) 2 ; 3

rewrite as $y = -2\cos \frac{\pi}{2}(x-2) + 3$

phase shift $= +2$

vert. disp $= +3$

3. Determine the period of $y = \tan\frac{\pi}{3}x$.

a) $\frac{1}{3}$
b) $\frac{2}{3}$
c) 3
d) 6

period for $y = \tan bx$ is π/b

$\therefore$ period $= \pi / \frac{\pi}{3} = 3$

4. Given a circle with radius 6 cm and length of arc 12 cm, determine the sector angle to the nearest degree.

a) $2°$
b) $29°$
c) $108°$
d) $115°$

$a = r\theta$

$12 = 6\theta$

$12/6 = \theta \longrightarrow \theta = 2$ rad

$2 \text{ rad} \times \frac{180°}{\pi \text{ rad}} = 114.59°$

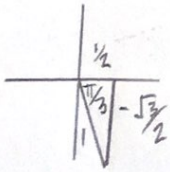
5. If $\csc x = -\frac{2}{\sqrt{3}}$ and $\tan x < 0$, determine $\cos x$.

a) $-\frac{\sqrt{3}}{2}$
b) $-\frac{1}{2}$
c) $\frac{1}{2}$
d) $\frac{\sqrt{3}}{2}$

① if $\csc x = -2/\sqrt{3}$ then $\sin x = -\sqrt{3}/2$ QUAD III or IV

② $\tan x < 0$ $\therefore$ QUAD II or IV $\therefore$ WE ARE IN QUAD IV

③



④ $\therefore \cos x = 1/2$

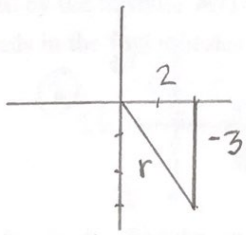
6. If $(2,-3)$ is on the terminal side of standard position angle θ, what is the value of $\sec\theta$?

a) $-\frac{\sqrt{13}}{2}$

b) $-\frac{\sqrt{13}}{3}$

c) $\frac{\sqrt{13}}{3}$

d) $\frac{\sqrt{13}}{2}$



① $2^2+(-3)^2=r^2$

$4+9=r^2$

$13=r^2$

$\sqrt{13}=r$

② $\sec\theta=\frac{H}{A}=\frac{r}{x}=\frac{\sqrt{13}}{2}$

7. Determine the smallest positive angle θ, in radians, such that $\csc\theta=-\sqrt{2}$

a) $\frac{\pi}{4}$

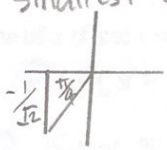
b) $\frac{3\pi}{4}$

c) $\frac{5\pi}{4}$

d) $\frac{7\pi}{4}$

① if $\csc\theta=-\sqrt{2}$ then $\sin\theta=\frac{-1}{\sqrt{2}}$ (quad. III or IV)

② smallest angle is in quad III

③ 

④ ref. angle is $\frac{\pi}{4}$ $\therefore \theta=\pi+\frac{\pi}{4}=\frac{4\pi}{4}+\frac{\pi}{4}=\frac{5\pi}{4}$

8. Convert 10 radians to a degree value between 0° and 360°.

a) 33°

b) 148°

c) 213°

d) 303°

① $10\text{ rad}\times\frac{180^\circ}{\pi\text{ rad}}=573^\circ$

② $573^\circ-360^\circ=213^\circ$ (coterminal to 573°)

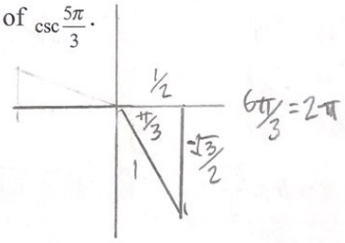
9. Determine the exact value of $\csc\frac{5\pi}{3}$.

a) $-\frac{2\sqrt{3}}{3}$

b) $-\frac{\sqrt{3}}{3}$

c) $\frac{\sqrt{3}}{3}$

d) $\frac{2\sqrt{3}}{3}$

① 

② $\csc=\frac{H}{O}=\frac{1}{-\frac{\sqrt{3}}{2}}=\frac{-2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$=\frac{-2\sqrt{3}}{3}$

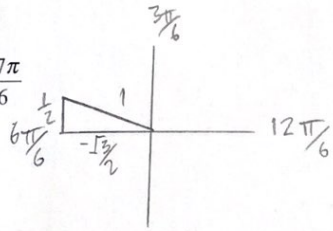
10. Determine the value of $\cot\frac{17\pi}{6}$

a) $-\sqrt{3}$

b) $-\frac{1}{\sqrt{3}}$

c) $\frac{1}{\sqrt{3}}$

d) $\sqrt{3}$

① 

② $\cot=\frac{A}{O}=\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}=-\sqrt{3}$

11. Determine the quadrant in which the terminal arm of θ lies for $\sec\theta<0$, $\tan\theta>0$ $\therefore$ QUAD I, III

a) quadrant I

b) quadrant II

c) quadrant III

d) quadrant IV

$\therefore \cos\theta<0$

QUAD II, III

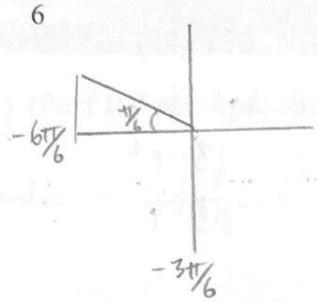
12. Determine the reference angle for $-\frac{7\pi}{6}$

a) $-\frac{\pi}{6}$

b) $\frac{\pi}{6}$

c) $\frac{5\pi}{6}$

d) $\frac{7\pi}{6}$



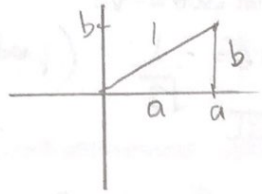
13. Determine $\sec\theta$ if the terminal arm of angle θ in standard position intersects the unit circle at point (a, b).

a) a

b) b

c) $\frac{1}{a}$

d) $\frac{1}{b}$



$\therefore \sec\theta = \frac{H}{A} = \frac{1}{a}$

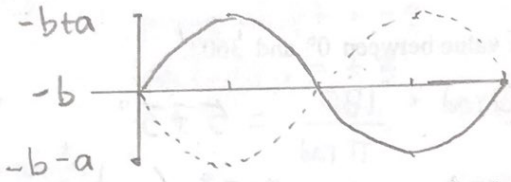
14. Determine the maximum value of $y = -a\sin x - b$, $a, b > 0$

a) $-a-b$

b) $-a+b$

c) $a-b$

d) $a+b$



15. Determine the minimum value of the function $y = a\sin x - b$, $a, b > 0$.

a) $-a-b$

b) $-a+b$

c) $a-b$

d) $a+b$

16. What is the value of $\tan B$ if $\cos B = -\frac{3}{5}$ and $\pi \le B < \frac{3\pi}{2}$?

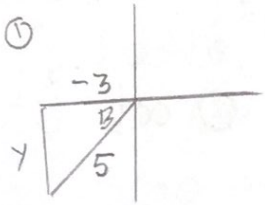
① QUAD III

a) $-\frac{4}{3}$

b) $-\frac{3}{4}$

c) $\frac{3}{4}$

d) $\frac{4}{3}$

① 

② $y = -4$ (Pythagorean Triple)

③ $\tan B = \frac{-4}{-3} = \frac{4}{3}$

17. Solve: $\sec x = \frac{-2\sqrt{3}}{3}$, $0 \le x < 2\pi$

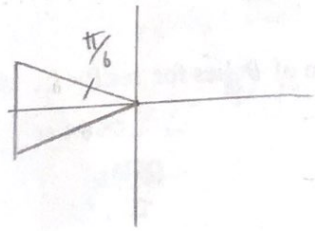
① $\cos x = \frac{3}{-2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{-2 \cdot 3} = -\frac{\sqrt{3}}{2}$

a) $\frac{2\pi}{3}, \frac{4\pi}{3}$

b) $\frac{2\pi}{3}, \frac{5\pi}{3}$

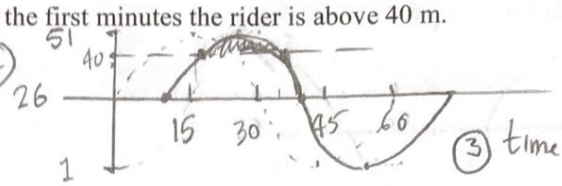
c) $\frac{5\pi}{6}, \frac{7\pi}{6}$

d) $\frac{5\pi}{6}, \frac{11\pi}{6}$

② 

18. The height, h, in metres, of a certain Ferris wheel seat above the ground at time, t, in seconds, after the ride is started is given by the formula $h(t) = 25\sin\frac{\pi}{30}(t-10)+26$. Use the graph of the function to determine the number of seconds in the first minutes the rider is above 40 m.

a) 15.7 sec.
b) 18.6 sec.
c) 26.3 sec.
d) 34.3 sec.

② 

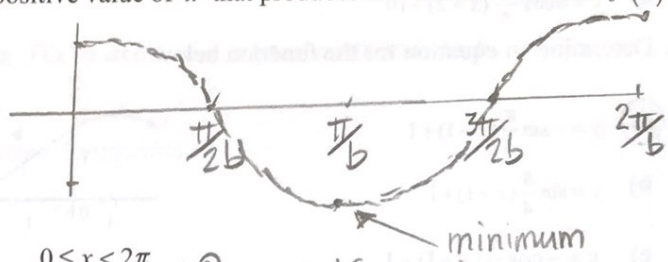
① Max = 26 + 25 = 51
Min = 26 − 25 = 1
period = $\frac{2\pi}{b} = \frac{2\pi}{\frac{\pi}{30}} = 60$

③ time above 40m ≐ 35 − 15 ≐ 20 sec

19. For $f(x) = \cos bx$, $b > 0$, find the smallest positive value of x that produces a minimum value for $f(x)$.

a) 0
b) $\frac{\pi}{2b}$
c) $\frac{\pi}{b}$
d) $\frac{3\pi}{2b}$

period = $\frac{2\pi}{b}$



20. Determine the values of x if $\csc x = -1.325$, $0 \le x < 2\pi$

a) 2.42, 4.00
b) 2.64, 3.88
c) 3.86, 5.55
d) 4.00, 5.43

if $\csc x = -1.325$

① $\sin x = -\frac{1}{1.325} = -0.7547$

② $x = \sin^{-1}(-0.7547)$
$x = -0.8552$ (ref. angle)

③ sin is negative in quad III & IV

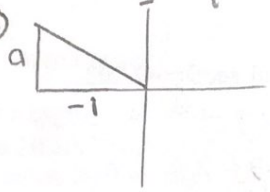
∴ $x = \pi + 0.8552 = 4.00$
$x = 2\pi - 0.8552 = 5.43$

21. Determine the value of $\csc x$ if $\tan x = -a$, where $a > 0$, $\cos x < 0$

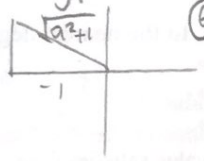
a) $\frac{-\sqrt{a^2+1}}{a}$
b) $\frac{\sqrt{a^2+1}}{a}$
c) $\frac{-\sqrt{a^2-1}}{a}$
d) $\frac{\sqrt{a^2-1}}{a}$

① tan is neg. in quad. II & IV

② cos is neg. in quad II & III ∴ angle is in quad II

③ 

④ Use Pythagorean Thm to find hypotenuse
$h^2 = a^2 + (-1)^2$
$h^2 = a^2 + 1$
$h = \sqrt{a^2+1}$

⑤ 

⑥ $\csc x = \frac{H}{O} = \frac{\sqrt{a^2+1}}{a}$

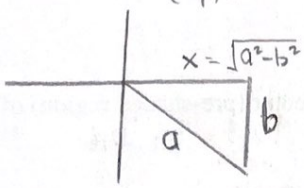
22. Determine the value of $\cos\theta$ if $\csc\theta = \frac{a}{b}$, where $\tan\theta < 0$ and $\sec\theta > 0$.

a) $\frac{\sqrt{a^2-b^2}}{a}$
b) $\frac{-\sqrt{a^2-b^2}}{a}$
c) $\frac{\sqrt{b^2-a^2}}{b}$
d) $\frac{-\sqrt{b^2-a^2}}{b}$

quad. II, IV ∴ $\cos\theta > 0$ quad I, IV

① QUAD IV

② if $\csc\theta = \frac{a}{b}$ (hyp) (opp)



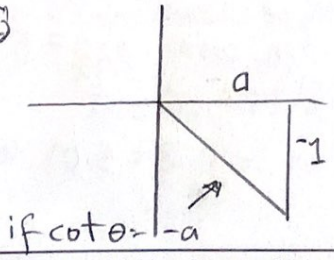
③ $x^2 + b^2 = a^2$
$x^2 = a^2 - b^2$
$x = \sqrt{a^2-b^2}$

④ $\cos\theta = \frac{\sqrt{a^2-b^2}}{a}$

23. Determine the value of $\sec\theta$ if $\cot\theta = -a$, where $a > 0$ and $\sin\theta < 0$.

a) $\frac{\sqrt{a^2+1}}{a}$
b) $-\frac{\sqrt{a^2+1}}{a}$
c) $\frac{a+1}{a}$
d) $-\frac{a+1}{a}$

quad. II & IV quad III & IV ∴ QUAD IV ①

② 

③ $HYP^2 = a^2 + (-1)^2$
$HYP = \sqrt{a^2+1}$

④ $\sec\theta = \frac{H}{A} = \frac{\sqrt{a^2+1}}{a}$

24. A cosine curve has a maximum point at $(2, 16)$ and the nearest minimum point to the right of this point is at $(7, 4)$. Which of the following is an equation for this curve?

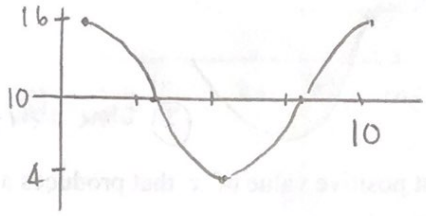
a) $y = 6\cos\frac{\pi}{5}(x-2)+10$

b) $y = 6\cos\frac{\pi}{5}(x+2)+10$

c) $y = 6\cos\frac{2\pi}{5}(x-2)+10$

d) $y = 6\cos\frac{2\pi}{5}(x+2)+10$

period $= 2(7-2) = 2(5) = 10 \longrightarrow 2\pi/10 = \pi/5$



phase shift is 2 units right $(x-2)$

25. Determine an equation for the function below

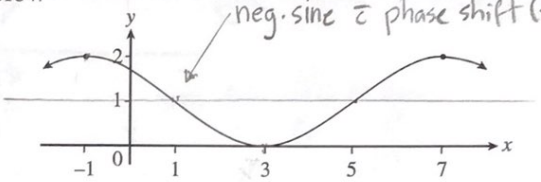
a) $y = -\sin\frac{\pi}{4}(x-1)+1$

b) $y = \sin\frac{\pi}{4}(x-1)+1$

c) $y = -\cos\frac{\pi}{4}(x+1)+1$

d) $y = \cos\frac{\pi}{4}(x-3)+1$

neg. sine & phase shift $(x-1)$



26. Evaluate $\cot\frac{2\pi}{5}$ $= \frac{1}{\tan 2\pi/5} = \frac{1}{3.078} = 0.325$

a) 0.203
b) 0.325
c) 1.021
d) 3.078

27. Find θ, to the nearest degree, if θ terminates in quadrant II and $\sec\theta = -2.202$.

a) 117°
b) 142°
c) 153°
d) no value satisfies the equation

if $\sec\theta = -2.202 \xrightarrow{①} \frac{1}{\cos\theta} = -2.202 \xrightarrow{②} \cos\theta = \frac{1}{-2.202}$

$\xrightarrow{③} \theta = \cos^{-1}\left(\frac{1}{-2.202}\right) \xrightarrow{④} \theta = 2.042$ rad

⑤ $2.042 \times 180/\pi = 117°$

28. The graph of the function $f(x) = \cos x$ is translated 3 units left. What is the equation of the shifted function?

a) $f(x) = \cos(x+3)$
b) $f(x) = \cos(x-3)$
c) $f(x) = \cos x + 3$
d) $f(x) = \cos x - 3$

replace x with $(x+3)$

29. A circle has a radius 10 cm. Determine the **area** of a sector (pre-shaded region) of the circle that has a central angle of 2.1 radians.

a) 21
b) 105
c) 150
d) 210

① Area $= \pi r^2 = \pi(10)^2 = 31.416$

② a full circle is 2π radians

③ $\frac{2.1}{2\pi} = 0.33$ ④ $0.33 \times 31.416 = 105$

30. Solve $\sec x = 3.45$, $0 \le x < 2\pi$.

a) 0.29, 2.85
b) 0.29, 5.99
c) 1.28, 1.86
d) 1.28, 5.01

① if $\sec x = 3.45$ then $\cos x = \frac{1}{3.45} = 0.2898$

② $\cos^{-1}(0.2898) = x$, $x = 1.28$ QUAD I

$x = 2\pi - 1.28 = 5.01$ QUAD IV

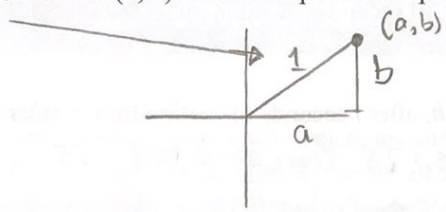
31. The point (a, b) is the point of intersection of the terminal arm of angle θ in standard position and the unit circle centred at (0, 0). Which expression represents $\csc\theta$?

a) $\frac{1}{a}$

b) a

c) $\frac{1}{b}$

d) b



$\therefore \csc\theta = \frac{H}{O} = \frac{1}{b}$

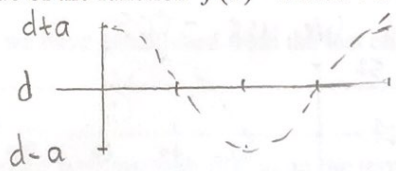
32. Determine the minimum value of the function $f(x) = a\cos x + d$, where $a > 0$ and $d > 0$.

a) $a - d$

b) $d - a$

c) $2a - d$

d) $d - 2a$



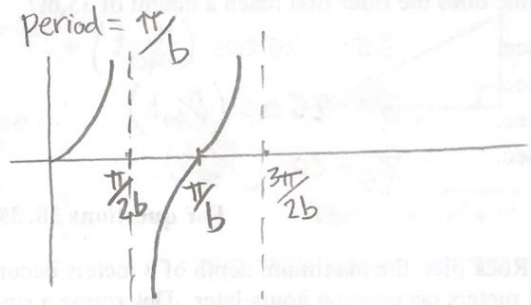
33. Determine the equations of the asymptotes of the function $y = \tan bx$, $b > 0$.

a) $x = \frac{n\pi}{b}$; n an integer

b) $x = \frac{n\pi}{2b}$; n an integer

c) $x = \frac{\pi}{b} + \frac{n\pi}{b}$; n an integer

d) $x = \frac{\pi}{2b} + \frac{n\pi}{b}$; n an integer



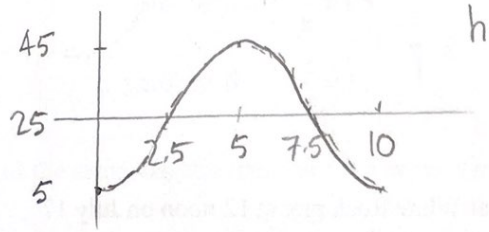
34. A wheel of radius 20 cm has its centre 25 cm above the ground. It rotates once every 10 seconds. Determine an equation for the height, h, above the ground of a point on the wheel at time t seconds if this point has a minimum height at $t = 0$ seconds.

a) $h = -20\cos\frac{\pi}{10}t + 5$

b) $h = -20\cos\frac{\pi}{5}t + 5$

c) $h = -20\cos\frac{\pi}{10}t + 25$

d) $h = -20\cos\frac{\pi}{5}t + 25$



$h = -20\cos\frac{2\pi}{10}t + 25$

35. The function $h(t) = 3.9\sin 0.16\pi(t-3) + 6.5$ gives the depth of water h metres, at any time, t hours, during a certain day. A freighter needs at least 8 metres of water to dock safely. How many hours in the 24-hour interval starting at $t = 0$ during which the freighter can dock safely?

a) 3.79

b) 4.68

c) 7.57

d) 9.36

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For questions 36 and 37

A Ferris wheel has a radius of 25 m and rotates every 80 seconds. A rider enters the seat at the lowest point of the Ferris wheel 2 metres above the ground.

36. Determine a sinusoidal function that gives the height h, after t seconds of motion for the rider

a) $h=-25\sin\left(\frac{\pi}{40}t\right)+27$

b) $h=25\sin\left(\frac{\pi}{40}t\right)+27$

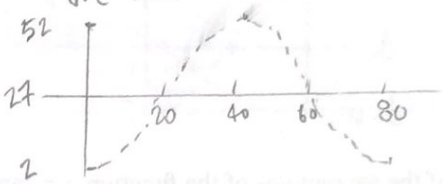
c) $h=-25\cos\left(\frac{\pi}{40}t\right)+27$

d) $h=25\cos\left(\frac{\pi}{40}t\right)+27$

• if radius = 25 then amplitude = 25

• period = 80 ∴ $b = \frac{2\pi}{80} = \frac{\pi}{40}$

• since rider starts at the lowest point we use − cos



37. At what time does the rider first reach a height of 35 m?

a) 4.1 sec.

b) 14.5 sec.

c) 24.1 sec.

d) 54.5 sec.

$35 = -25\cos\left(\frac{\pi}{40}t\right)+27$

$8 = -25\cos\left(\frac{\pi}{40}t\right)$

$-\frac{8}{25} = \cos\left(\frac{\pi}{40}t\right)$

$\cos^{-1}\left(\frac{-8}{25}\right) = \frac{\pi}{40}t$

$1.8965 = \frac{\pi}{40}t$

$\frac{40(1.8965)}{\pi} = t$

$t = \boxed{24.1 \text{ sec}}$

For questions 38, 39 and 40

38. At White Rock pier, the maximum depth of 8 meters occurred at 4:00 a.m. on July 1, with the minimum depth of 2 meters occurring 6 hours later. Determine a sinusoidal curve in terms of sine for this function.

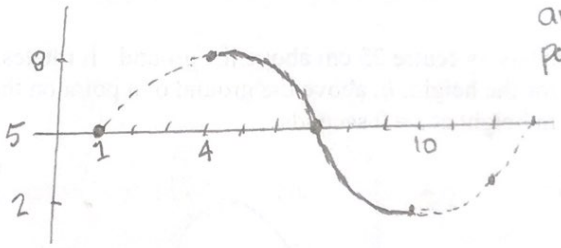
a) $d(t)=-5\sin\frac{\pi}{6}(t-1)+3$

b) $d(t)=-3\sin\frac{\pi}{6}(t-1)+5$

c) $d(t)=3\sin\frac{\pi}{6}(t-1)+5$

d) $d(t)=5\sin\frac{\pi}{6}(t-1)+3$

amp. = 3
positive sin



39. What was the depth of water at White Rock pier at 12 noon on July 1?

a) 2.5 meters

b) 3 meters

c) 3.5 meters

d) 4 meters

$d = 3\sin\frac{\pi}{6}(12-1)+5$

$d = 3(0.5)+5$

$d = -1.5+5 = 3.5$

40. Determine the first time on July 1 that the water reached a depth of 7 meters.

a) 2:24 a.m.

b) 2:39 a.m.

c) 5:22 a.m.

d) 5:36 a.m.

$7 = 3\sin\frac{\pi}{6}(t-1)+5$

$2 = 3\sin\frac{\pi}{6}(t-1)$

$\frac{2}{3} = \sin\frac{\pi}{6}(t-1)$

$\sin^{-1}\left(\frac{2}{3}\right) = \frac{\pi}{6}(t-1)$

$0.7297 = \frac{\pi}{6}(t-1)$

$\frac{6(0.7297)}{\pi} = t-1$

$1.394 = t-1$

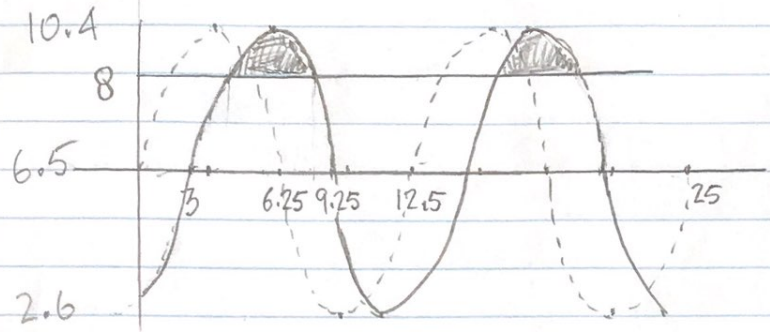
$t = 2.394$

$.394 \times 60$ min
$= 24$ min

P.297

#35 $h(t) = 3.9 \sin 0.16\pi(t-3) + 6.5$

amplitude = 3.9
v.d. = 6.5
MAX = 6.5 + 3.9 = 10.4
MIN = 6.5 − 3.9 = 2.6
period = $\frac{2\pi}{0.16\pi}$ = 12.5



$8 = 3.9 \sin 0.16\pi(t-3) + 6.5$

$1.5 = 3.9 \sin 0.16\pi(t-3)$

$\frac{1.5}{3.9} = \sin 0.16\pi(t-3)$

$\sin^{-1}\left(\frac{1.5}{3.9}\right) = 0.16\pi(t-3)$

$0.395 = 0.16\pi(t-3)$

$\frac{0.395}{0.16\pi} = t-3$

$0.79 = t-3 \longrightarrow t = 3.79$

* the interval when the water is 8m is between 3.79 and 9.25 − 0.79 which is 9.25 − 0.79 − 3.79 = 4.67 hours

This occurs twice in 24 hrs ∴ 4.67 × 2 = 9.34 hrs (d)