Pre-Calculus 12
Ch.6 Test - Trigonometry (Part II)

Name: _____
Score: _____

Practice Test

d 1. Solve over the set of real numbers. (1 mark)

period = 2π (eliminate a and b)

$2\sin^2 x - 5\sin x - 3 = 0$

a. $\frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$, n is an integer

b. $\frac{7\pi}{6} + n\pi, \frac{11\pi}{6} + n\pi$, n is an integer

c. $\frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n$, n is an integer

d. $\frac{7\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n$, n is an integer

let $\sin x = a$

$\therefore 2a^2 - 5a - 3 = 0$

$(2a+1)(a-3) = 0$

$\therefore (2\sin x + 1)(\sin x - 3) = 0$

$\boxed{2\sin x + 1 = 0}$ or $\sin x - 3 = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$

$\sin x = 3$ —— NO SOLN.

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

d 2. Determine the restriction for the expression $\frac{\tan\theta}{2\cos\theta - 1}$ (1 mark)

a. $\cos\theta \neq \frac{1}{2}$

b. $\sin\theta \neq 0$

c. $\sin\theta \neq 0, \cos\theta \neq \frac{1}{2}$

d. $\cos\theta \neq 0, \cos\theta \neq \frac{1}{2}$

$= \dfrac{\frac{\sin x}{\cos x}}{2\cos x - 1}$ ← $\cos x \neq 0$

$2\cos x - 1 \longleftrightarrow 2\cos x - 1 \neq 0 \longrightarrow \cos x \neq \frac{1}{2}$

b 3. Determine an equivalent expression to $\sin(2x - \pi)$. (1 mark)

a. $2\sin x\cos$

b. -2sinxcosx

c. $\cos^2 x - \sin^2 x$

d. $\sin^2 x - \cos^2 x$

$\sin(2x - \pi) = \sin 2x \cos\pi - \cos 2x \sin\pi$

$= \sin 2x\,(-1) - \cos 2x\,(0)$

$= -\sin 2x$

$= -2\sin x\cos x$

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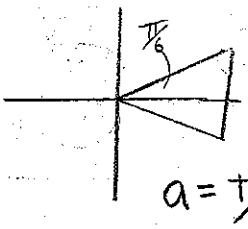
For #4 and #5, solve $0 \le x < 2\pi$.

4. $\cos 2x = \frac{\sqrt{3}}{2}$ (2 marks)

let $2x = a$

$\therefore \cos a = \frac{\sqrt{3}}{2}$



$a = \pi/6 \ , \ 11\pi/6$

but, $2x = a$

$\therefore 2x = \pi/6 \qquad 2x = 11\pi/6$

$x = \pi/12 \qquad x = 11\pi/12$

PERIOD $= \frac{2\pi}{2} = \pi$

$x = \pi/12 + \pi = 13\pi/12$

$x = 11\pi/12 + \pi = 23\pi/12$

x: $\pi/12, 11\pi/12, 13\pi/12, 23\pi/12$

5. $2\cos^2 x - \sin x - 1 = 0$ (3 marks)

identity: $\boxed{2\cos^2 x - 1 = \cos 2x}$

$\therefore \cos 2x - \sin x = 0$

identity: $\boxed{\cos 2x = 1 - 2\sin^2 x}$

$\therefore 1 - 2\sin^2 x - \sin x = 0$

$-2\sin^2 x - \sin x + 1 = 0$

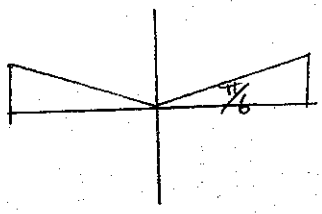
$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\therefore 2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$2\sin x = 1 \qquad \sin x = -1$

$\sin x = 1/2 \qquad \boxed{x = 3\pi/2}$



$\boxed{x = \pi/6}$

$x = \pi - \pi/6$

$\boxed{= 5\pi/6}$

x: $\pi/6, 5\pi/6, 3\pi/2$

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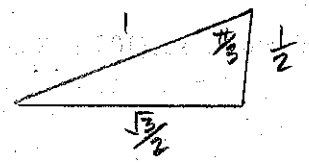
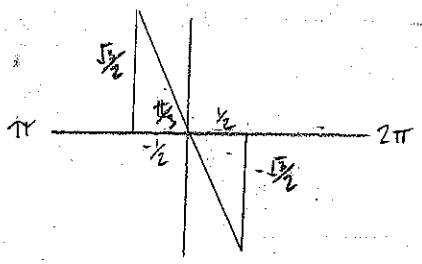
For #6 and #7
i) Solve $0 \le x < 2\pi$. (2 marks)
ii) Also give the general form. (1 mark)



6. $\cot x = -\frac{1}{\sqrt{3}}$

$\therefore \tan x = -\sqrt{3}$



$x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Period $= \frac{\pi}{b} = \frac{\pi}{1} = \pi$

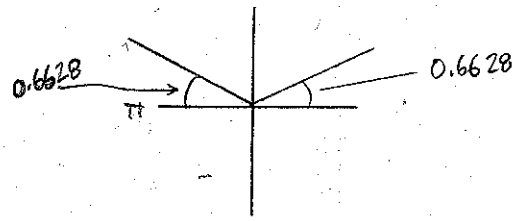
$b = 1$

x: $\frac{2\pi}{3}, \frac{5\pi}{3}$

general form: $x = \frac{2\pi}{3} + n\pi$ where n is an integer

7. $\csc x = 1.6251$

$\frac{1}{\sin x} = 1.6251$



$\therefore \sin x = (1.6251)^{-1}$

$\sin x = 0.6153$

$x = \sin^{-1}(0.6153)$

$x = \boxed{0.6628 \text{ radians}}$

$x = \pi - 0.6628$

$\boxed{x = 2.4788}$

x: 0.6628, 2.4788

general form: $x = 0.6628 + 2\pi n$

$x = 2.4788 + 2\pi n$

where n is an integer

period $= 2\pi$

8. Solve for $-2\pi \le x \le 2\pi$. (3 marks)

$5\sin^2 x + 3\sin x = 2$

let $\sin x = a$

$\therefore 5a^2 + 3a - 2 = 0$

$a^2 + 3a - 10$

$(a+5)(a-2)$

$\left(a + \frac{5}{5}\right)\left(a - \frac{2}{5}\right)$

$(a+1)(5a-2) = 0$

but $a = \sin x$

$\therefore (\sin x + 1)(5\sin x - 2) = 0$

$\therefore \sin x + 1 = 0$ or $5\sin x - 2 = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$

$5\sin x = 2$

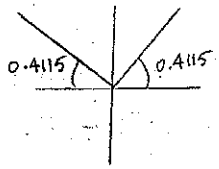
$\sin x = \frac{2}{5}$

$x = \sin^{-1}\frac{2}{5}$

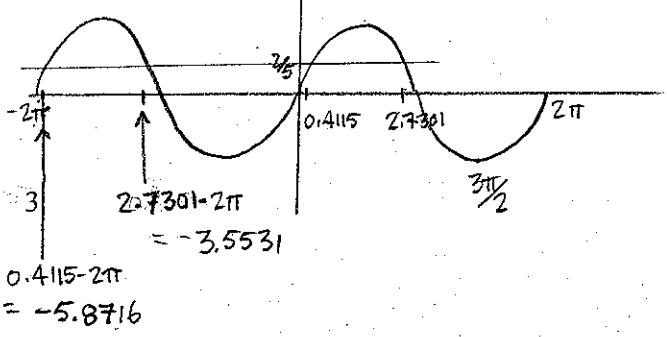
$x = 0.4115$

$x = \pi - 0.4115$

$= 2.7301$



$\boxed{x = -5.8716, -3.5531, 0.4115, 2.7301, -\frac{\pi}{2}, \frac{3\pi}{2}}$



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9. Find the exact value of sec(105°). (3 marks)

Since 105° is NOT an angle found on a useful triangle, we must look at the question as

$\sec(45° + 60°)$

$$= \frac{1}{\cos(45° + 60°)} = \frac{1}{\cos 45 \cos 60 - \sin 45 \sin 60}$$

sec(105°): $-\sqrt{2} - \sqrt{6}$

$$= \frac{1}{\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}} = \frac{1}{\frac{\sqrt{2} - \sqrt{6}}{4}} = \frac{4}{\sqrt{2} - \sqrt{6}} \times \frac{\sqrt{2} + \sqrt{6}}{\sqrt{2} + \sqrt{6}} = \frac{4(\sqrt{2} + \sqrt{6})}{2 - 6}$$

$$= \frac{4(\sqrt{2} + \sqrt{6})}{-4} = -(\sqrt{2} + \sqrt{6})$$

10. Prove the identity: $\frac{\cos x}{1 - \sin x} = \sec x + \tan x$ (3 marks)

LEFT SIDE	RIGHT SIDE
	$\sec x + \tan x$
	$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$
	$= \frac{1 + \sin x}{\cos x}$
	$= \frac{1 + \sin x}{\cos x} \times \frac{1 - \sin x}{1 - \sin x}$
	$= \frac{1 - \sin^2 x}{\cos x (1 - \sin x)}$
	$= \frac{\cos^2 x}{\cos x (1 - \sin x)}$
	$= \frac{\cos x}{1 - \sin x}$

L.S. = R.S.

Q.E.D.

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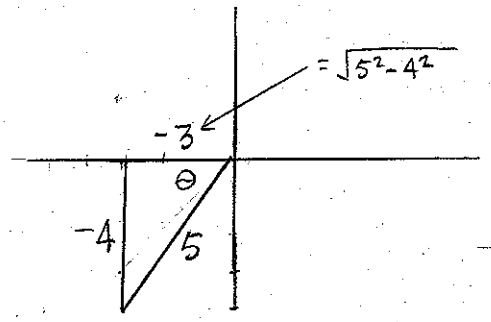
11. If $\sin x = -\frac{4}{5}$ in Quadrant III, then find $\sin 2x$. Give an exact answer, using fractions. (3 marks)



$\sin 2x = 2 \sin x \cos x$

$= 2\left(\frac{-4}{5}\right)\left(\frac{-3}{5}\right)$

$= \frac{24}{25}$

$\cos\theta = \frac{-3}{5}$

$\sin 2x$: $\frac{24}{25}$

12. Prove the identity: $\frac{1-\cos x}{\sin^2 x} = \frac{1}{1+\cos x}$ (3 marks)

LEFT SIDE	RIGHT SIDE
	$= \frac{1}{1+\cos x} \times \frac{1-\cos x}{1-\cos x}$
	$= \frac{1-\cos x}{1-\cos^2 x}$
	$= \frac{1-\cos x}{\sin^2 x}$

L.S. = R.S.

Q.E.D.

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13. Prove the identity: $\frac{\cot x}{\csc x - 1} = \frac{\csc x + 1}{\cot x}$ (3 marks)

LEFT SIDE	RIGHT SIDE
	$= \frac{\csc x + 1}{\cot x} \times \frac{\csc x - 1}{\csc x - 1}$
	$= \frac{\csc^2 x - 1}{\cot x(\csc x - 1)}$
	$= \frac{\cot^2 x}{\cot x(\csc x - 1)}$
	$= \frac{\cot x}{\csc x - 1}$

L.S. = R.S

Q.E.D.

14. Solve the equation, giving exact values for x, where $0 \le x < 2\pi$. (4 marks) Also, give the general solution for the equation. (1 mark)

period = 2π

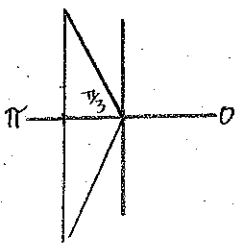
$2\cos^2 x + 3\cos x + 1 = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$2\cos x + 1 = 0$ or $\cos x + 1 = 0$

$2\cos x = -1$ $\quad$ $\cos x = -1$

$\cos x = -\frac{1}{2}$ $\quad$ $x = \pi$



$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

x: $\frac{2\pi}{3}, \pi, \frac{4\pi}{3}$

general solution: _____

$x = \frac{2\pi}{3} + 2\pi n$

$x = \pi + 2\pi n$

$x = \frac{4\pi}{3} + 2\pi n$, where n is an integer

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15. Prove the identity: $\frac{2\cos x + 2\cos^2 x}{\sin 2x} = \frac{\sin x}{1-\cos x}$ (5 marks)

LEFT SIDE	RIGHT SIDE
$\frac{2\cos x + 2\cos^2 x}{\sin 2x}$	
$= \frac{\cancel{2\cos x}(1+\cos x)}{\cancel{2}\sin x \cancel{\cos x}}$	
$= \frac{1+\cos x}{\sin x} \times \frac{1-\cos x}{1-\cos x}$	
$= \frac{1-\cos^2 x}{\sin x(1-\cos x)}$	
$= \frac{\sin^{\cancel{2}} x}{\cancel{\sin x}(1-\cos x)}$	
$= \frac{\sin x}{1-\cos x}$	L.S. = R.S. Q.E.D.

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16. Prove the identity: $\cos 2x = \dfrac{\cot x - \sin 2x}{\cot x}$ (5 marks)

LEFT SIDE	RIGHT SIDE
	$= \dfrac{\dfrac{\cos x}{\sin x} - 2\sin x\cos x}{\dfrac{\cos x}{\sin x}}$
	$= \dfrac{\dfrac{\cos x}{\sin x} - \dfrac{2\sin^2 x\cos x}{\sin x}}{\dfrac{\cos x}{\sin x}}$
	$= \dfrac{\dfrac{\cos x - 2\sin^2 x\cos x}{\sin x}}{\dfrac{\cos x}{\sin x}}$ these cancel out
	$= \dfrac{\cos x - 2\sin^2 x\cos x}{\cos x}$
	$= \dfrac{\cancel{\cos x}(1-2\sin^2 x)}{\cancel{\cos x}}$
	$= 1 - 2\sin^2 x$
	$= \cos 2x$

L.S. = R.S.

Q.E.D.