Pre-Calculus 12
Ch. 4 Practice Test
Logs and Exponents

Name KEY

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Section A: (Non- Calculator)

1. Which expression is equivalent to $\log\left(\frac{100m^3}{n^2}\right)$? (no calculator permitted)

A. $6 \log m - 2 \log n$
B. $2 + \log 3m - \log 2n$
C. $2 + 3 \log m - 2 \log n$
D. $100 + 3 \log m - 2 \log n$

$= \log 100m^3 - \log n^2$
$= \log 100 + \log m^3 - 2\log n$
$= 2 + 3\log m - 2\log n$

2. Change to exponential form $\log_3 81 = 4$ (no calculator permitted)

A. $4^3 = 81$
B. $81^{-3} = 4$
C. $3^4 = 81$
D. $81^4 = 3$

use definition of a log

3. Express as a single logarithm $3 \log A - 2 \log B - \log C$ (no calculator permitted)

A. $\log\left(\frac{A^3}{B^2C}\right)$
B. $\log\left(\frac{A^3C}{B^2}\right)$
C. $\log(A^3 - B^2 - C)$
D. $\log(3A - 2B - C)$

$= \log A^3 - \log B^2 - \log C$
$= \log\left(\frac{A^3}{B^2}\right) - \log C$
$= \log\left(\frac{\frac{A^3}{B^2}}{C}\right)$
$= \log\left(\frac{A^3}{B^2C}\right)$

$\frac{\frac{A^3}{B^2}}{C} = \frac{A^3}{B^2} \div C$
$= \frac{A^3}{B^2} \times \frac{1}{C} = \frac{A^3}{B^2C}$

4. Determine the equation of the asymptote: $y = 5 \log (x - 3) + 2$ (no calculator permitted)

A. $x = 2$
B. $x = -2$
C. $x = 3$
D. $x = -3$

$x - 3 > 0$
$\therefore x > 3$ which means the asymptote is at $x = 3$

5. Solve for x : $9^{x-1} = \frac{1}{27}$ (no calculator permitted)

A. $\frac{7}{2}$

B. $-\frac{1}{2}$

C. -1

D. $\frac{2}{3}$

$(3^2)^{x-1} = \frac{1}{3^3}$

$3^{2x-2} = 3^{-3}$ since the bases are = the exponents must be =

$\therefore \; 2x-2=-3$

$2x=-3+2$

$2x=-1$

$x=-1/2$

6. Solve for x: $2\log_3(-x) = 2 - \log_3 4$ (no calculator permitted)

A. $\frac{3}{2}$

B. $-\frac{3}{2}$

C. -1

D. $\pm\frac{3}{2}$

$\log_3(-x)^2 = 2 - \log_3 4$

$\log_3(-x)^2 + \log_3 4 = 2$

$\log_3(x^2)(4) = 2$

apply def^{n} of a log

$4x^2 = 3^2 \longrightarrow x^2 = \frac{9}{4} \longrightarrow x = \pm\sqrt{\frac{9}{4}} \longrightarrow x = \pm\frac{3}{2}$

$x = \frac{-3}{2}, \cancel{\frac{3}{2}}$ EXTRANEOUS

7. Evaluate: $2\log_b\left(\frac{1}{b^3}\right)$ (no calculator permitted)

A. -6

B. $-\frac{2}{3}$

C. $\frac{2}{3}$

D. 6

$= \log_b\left(\frac{1}{b^3}\right)^2 = \log_b(b^{-3})^2 = \log_b b^{-6} = -6$

Section B: **(Calculator permitted)**

8. Evaluate : $\log_5 49$

$= \dfrac{\log 49}{\log 5} = 2.418 = 2.42$

A. 2.41
B. 2.42
C. 0.34
D. 0.99

9. Solve: $\log_5(1-x) - \log_5(4-2x) = \log_5 3$

$\log_5\left(\dfrac{1-x}{4-2x}\right) = \log_5 3$

A. −2
B. 2.2
C. 6
D. no solution

$\therefore \dfrac{1-x}{4-2x} = 3 \longrightarrow 1-x = 3(4-2x) \longrightarrow 1-x = 12-6x$

$\longrightarrow 6x - x = 12 - 1 \longrightarrow 5x = 11 \longrightarrow x = \dfrac{11}{5} = 2.2$

CHECK $\log_5(1-2.2) = \log_5(-1.2)$ $\therefore$ 2.2 is extraneous

10. What is the equation of the inverse of the exponential function $y = 4^x$?

A. $y = \log_4 x$
B. $y = \log_x 4$
C. $x = \log_4 y$
D. $x = \log_y 4$

$x = 4^y$

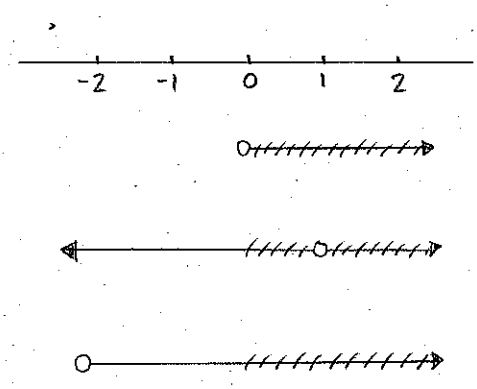
apply def^n of a log

$y = \log_4 x$

11. Give the domain of $y = \log_x(x+2)$

A. $x > -2$
B. $x > 0$
C. $x > 0$, $x \neq 1$
D. $x > -2$, $x \neq 1$

restrictions on base $\begin{cases} x > 0 \\ x \neq 1 \end{cases}$



GRAPH OVERLAP:

$x > 0$, $x \neq 1$

$x + 2 > 0 \Longrightarrow x > -2$

12. Determine the value of $\log 100C^3$ if $\log C = 2$.

A. 5

B. 6

C. 7

D. 8

$= \log 100 + \log C^3$

$= 2 + 3\log C$ but, $\log C = 2$

$\therefore = 2 + 3(2)$

$= 2 + 6 = 8$

13. Express as a single logarithm: $3 - \log b + \log c$

A. $\log \frac{3}{bc}$

B. $\log \frac{3c}{b}$

C. $\log \frac{1000}{bc}$

D. $\log \frac{1000c}{b}$

$3 = \log 1000$

$\therefore \log 1000 - \log b + \log c$

$= \log\left(\frac{1000}{b}\right) + \log c$

$= \log\left(\frac{1000c}{b}\right)$

14. Sue invests \$2000 in a bond which pays an interest rate of 8% per annum compounded semi-annually. After 3 years the what is the value of the bond?

A. \$2249.74

B. \$2519.42

C. \$2530.64

D. \$3173.75

$A = P\left(1 + \frac{0.08}{2}\right)^{2(3)}$

$A = 2000(1.04)^6$

$A = \$2530.64$

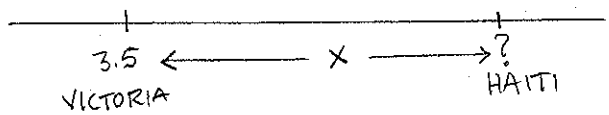
15. An earthquake in Victoria measured 3.5 on the Richter scale. An earthquake in Haiti was 2000 times as intense. What was the Richter scale reading of the earthquake in Haiti?

A. 3.3

B. 5.5

C. 6.8

D. 7.3



$10^x = 2000 \longrightarrow \log 2000 = x$ - by applying def^n of a log

$\therefore x = 3.3$

$3.5 + x = ? \longrightarrow 3.5 + 3.3 = ? \longrightarrow ? = 6.8$

Section C: (Written)

16. If $\log_3 2 = m$ and $\log_3 7 = n$, determine an expression for $\log_3 \frac{12}{49}$ in terms of m and n. (2 marks)

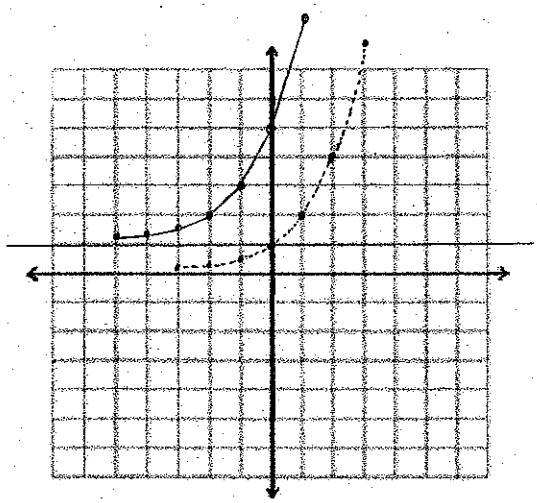
$$\log_3 \frac{12}{49} = \log_3 12 - \log_3 49 = \log_3 (4\times 3) - \log_3 7^2 = \log_3 4 + \log_3 3 - 2\log_3 7$$

$$= \log_3 2^2 + 1 - 2n = 2\log_3 2 + 1 - 2n = \boxed{2m + 1 - 2n}$$

$\log_3 3 = 1$ $\quad$ $\log_3 7 = n$ $\quad$ $\log_3 2 = m$

17. Sketch the graph of $y = 2^{x+2} + 1$ and answer the following: (3 marks)

x-int: $\varnothing$ y-int: $(0,5)$ Domain: x = all real #'s Range: $y > 1$



base function

$y = 2^x$

x	y
-3	1/8
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8

$y = 2^{x+2} + 1$ ← horiz. trans 2 units LEFT; ↖ vert. trans. 1 unit UP

x	y
-5	1 1/8
-4	1 1/4
-3	1 1/2
-2	2
-1	3
0	5
1	9

18. Solve for x algebraically (no calculator) : $\frac{4^{x-3}}{16^x} = 8^{x-1}$ (3 marks)

$$\frac{(2^2)^{x-3}}{(2^4)^x} = (2^3)^{x-1} \longrightarrow \frac{2^{2x-6}}{2^{4x}} = 2^{3x-3} \longrightarrow 2^{2x-6-4x} = 2^{3x-3}$$

$$\longrightarrow 2^{-2x-6} = 2^{3x-3} \xrightarrow[\text{bases are equal}]{} \therefore -2x-6 = 3x-3$$

$$-2x-3x = 6-3$$

$$-5x = 3$$

$$\boxed{x = \frac{-3}{5}}$$

19. A strain of bacteria triples every 5 days. How long will it take for an initial amount of 200 bacteria to grow to 25 000 bacteria?
(Solve algebraically using logarithms. Answer accurately to 2 decimal places.) (4 marks)

$$25000 = 200\,(3)^{x/5}$$

$$\frac{25000}{200} = 3^{x/5}$$

$$125 = 3^{x/5}$$

apply def^n of a log

$$\log_3 125 = \frac{x}{5} \longrightarrow \frac{\log 125}{\log 3} = \frac{x}{5} \longrightarrow \frac{5\log 125}{\log 3} = x \longrightarrow \boxed{x = 21.97 \text{ days}}$$

20. Solve algebraically: $\log_2(2x + 12) - \log_2(x - 3) = 3$ (4 marks)

$$\log_2\left(\frac{2x+12}{x-3}\right) = 3$$

apply def^n of a log

$$2^3 = \frac{2x+12}{x-3}$$

$$(x-3)\,8 = 2x+12$$

$$8x-24 = 2x+12$$

$$8x-2x = 12+24 \longrightarrow 6x = 36 \longrightarrow \boxed{x = 6}$$

21. Solve algebraically: $\log_6(2x + 1) + \log_6 x = 1$ (4 marks)

$$\log_6 (2x+1)(x) = 1$$

apply def^n of a log

$$6^1 = (2x+1)(x) \longrightarrow 6 = 2x^2+x \longrightarrow 0 = 2x^2+x-6$$

$$\longrightarrow (2x-3)(x+2) = 0 \longrightarrow 2x-3=0 \text{ or } x+2=0$$

$$2x = 3 \qquad \boxed{\cancel{x=-2}}$$

$$\boxed{x = 3/2}$$

EXTRANEOUS