

$$5 \text{ a) } \log_b x^{\log_x a} \xrightarrow[\text{POWER RULE}]{} = (\log_x a) \log_b x$$

$$\xrightarrow[\text{CHANGE OF BASE}]{} = \frac{\log a}{\log x} \times \frac{\log x}{\log b}$$

$$= \frac{\log a}{\log b}$$

$$\xrightarrow[\text{CHANGE OF BASE}]{} = \log_b a$$

$$\begin{aligned} \text{b) } x^{\log_x 20 - \log_x 4} &= x^{\log_x \left(\frac{20}{4}\right)} && \text{QUOTIENT RULE} \\ &= x^{\log_x 5} \\ &= x^{\log_x 5} \end{aligned}$$

let $x^{\log_x 5} = y$ and apply the def'n of a log

$$\therefore \log_x y = \log_x 5 \xrightarrow{} \therefore y = 5$$

$$\text{and } x^{\log_x 5} = 5$$

$$\text{c) } (\log_2 10)(\log 48 - \log 3) = \frac{\log 10}{\log 2} \left(\log \frac{48}{3} \right)$$

$$= \frac{1}{\log 2} (\log 16)$$

$$= \frac{1}{\log 2} (\log 2^4)$$

$$= \frac{4 \log 2}{\log 2} = 4$$

$$5 \text{ d) } \frac{\log x^3 + \log x^5}{\log x^6 - \log x^3} = \frac{\log(x^3 \cdot x^5)}{\log(\frac{x^6}{x^3})} = \frac{\log x^8}{\log x^2}$$

$$= \frac{8 \log x}{2 \log x} = 4$$

$$\text{e) } \left(\frac{a}{b}\right)^{\log 0.5} \cdot \left(\frac{a}{b}\right)^{\log 0.2} = \left(\frac{a}{b}\right)^{\log 0.5 + \log 0.2}$$

$$= \left(\frac{a}{b}\right)^{\log(0.5 \cdot 0.2)}$$

$$= \left(\frac{a}{b}\right)^{\log 0.1}$$

$$\boxed{\begin{aligned} \log 0.1 &= \log 10^{-1} \\ &= -1 \log 10 \\ &= -1 \end{aligned}} \quad = \left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)$$

$$9) 10 \log_4 x - 12 \log_8 x = \frac{10 \log x}{\log 4} - \frac{12 \log x}{\log 8}$$

$$= \frac{10 \log x}{\log 2^2} - \frac{12 \log x}{\log 2^3}$$

$$= \frac{10 \log x}{2 \log 2} - \frac{12 \log x}{3 \log 2}$$

$$= \frac{5 \log x}{\log 2} - \frac{4 \log x}{\log 2} = \frac{\log x - \log x}{\log 2}$$

$$5i) \log(1-x^3) - \log(1+x+x^2) - \log(1-x)$$

$$= \log\left(\frac{1-x^3}{1+x+x^2}\right) - \log(1-x)$$

$$= \log\left(\frac{1-x^3}{(1+x+x^2)(1-x)}\right) = \log\left(\frac{1-x^3}{1-x^3}\right) = \log 1 = 0$$

$$5k) \frac{1}{\log_a x} + \frac{1}{\log_b x} = \frac{1}{\frac{\log x}{\log a}} + \frac{1}{\frac{\log x}{\log b}}$$

$$= \frac{\log a}{\log x} + \frac{\log b}{\log x}$$

$$= \frac{\log a + \log b}{\log x}$$

$$= \frac{\log ab}{\log x}$$

$$= \log_x ab$$