FMPC 12 - 5.1 EXERCISE SET QUESTIONS 1 AND 2

1 a) $\dfrac{(3^{1/5})^{10} \cdot (3^{-3})}{9} = \dfrac{(3^{10/5})(3^{-3})}{3^2}$ because #3 $(a^m)^n = a^{mn}$

$9 = 3^2$

$= \dfrac{\cancel{(3^2)}(3^{-3})}{\cancel{3^2}}$

$= 3^{-3} = \dfrac{1}{3^3}$ because #6 $a^{-m} = \dfrac{1}{a^m}$

$= \boxed{\dfrac{1}{27}}$

b) $\dfrac{(-4x^2y^{-2})^{-3}}{x^{-1}y^2} = \dfrac{-4^{-3}x^{-6}y^6}{x^{-1}y^2}$ because #3 $(a^m)^n = a^{mn}$

$= \dfrac{x\ y^6}{-4^3x^6y^2}$ because #6 $a^{-m} = \dfrac{1}{a^m}$

$= \boxed{\dfrac{y^4}{-64\ x^5}}$ because #2

c) $$\frac{125^{3x-1} \cdot 25^{1-2x}}{\left(\frac{1}{5}\right)^{2x-3}} = \frac{\left(5^3\right)^{3x-1} \cdot \left(5^2\right)^{1-2x}}{\left(5^{-1}\right)^{2x-3}}$$

$$= \frac{5^{9x-3} \cdot 5^{2-4x}}{5^{-2x+3}}$$

$$= \frac{5^{9x-3+2-4x}}{5^{-2x+3}}$$

$$= \frac{5^{5x-1}}{5^{-2x+3}} = 5^{5x-1-(-2x+3)}$$

$$= 5^{5x-1+2x-3}$$

$$= \boxed{5^{7x-4}}$$

d) $$\frac{2x^4 \cdot 3^{5x} - 4x^3 \cdot 3^{5x}}{x^3 - 2x^2} = \frac{3^{5x}\left(2x^4 - 4x^3\right)}{x^2(x-2)}$$

① factor $3^{5x}$ from numerator

② factor $x^2$ from denominator

③ factor $2x^3$ out of $(2x^4 - 4x^3)$

④ cancel $(x-2)$ and $x^2$

$$= \frac{3^{5x} \cdot 2x^{\cancel{3}}\cancel{(x-2)}}{\cancel{x^2}\cancel{(x-2)}}$$

$$= \boxed{3^{5x} \cdot 2x}$$

e) $\left(4^{-x} \cdot 8^{x}\right)^{2} = \left(\left(2^{2}\right)^{-x} \cdot \left(2^{3}\right)^{x}\right)^{2}$ because $4 = 2^{2}$, $8 = 2^{3}$

$$= \left(2^{-2x} \cdot 2^{3x}\right)^{2}$$

$$= \left(2^{-2x+3x}\right)^{2}$$

$$= \left(2^{x}\right)^{2} = 2^{2x} = \left(2^{2}\right)^{x} = \boxed{4^{x}}$$

f) $\dfrac{2^{x}\left(2^{x}+2^{-x}\right) - 2^{x}\left(2^{x}-2^{-x}\right)}{2^{-2}}$

factor out $2^{x}$ in numerator

$$= \frac{2^{x}\left(\left(2^{x}+2^{-x}\right) - \left(2^{x}-2^{-x}\right)\right)}{2^{-2}}$$

$$= \frac{2^{x}\left(\cancel{2^{x}} + 2^{-x} - \cancel{2^{x}} + 2^{-x}\right)}{2^{-2}}$$

$$= \frac{2^{x}\left(2^{-x}+2^{-x}\right)}{2^{-2}}$$ = expand the numerator

$$= \frac{\left(2^{x}\right)\left(2^{-x}\right) + \left(2^{x}\right)\left(2^{-x}\right)}{2^{-2}} = \frac{2^{x+(-x)} + 2^{x+(-x)}}{2^{-2}}$$

$$= \frac{2^{0}+2^{0}}{2^{-2}} = \frac{1+1}{2^{-2}} = \frac{2}{2^{-2}} = 2 \times 2^{2} = 2^{3} = \boxed{8}$$

2. a) $4^{x^2-x} = 1$ remember, $4^0 = 1$

$\therefore 4^{x^2-x} = 4^0$ since the bases are equal, the exponents MUST be equal

$\therefore x^2 - x = 0$

$x(x-1) = 0 \longrightarrow$ this means either

$\boxed{x = 0}$ OR $x - 1 = 0$

$\boxed{x = 1}$

b) $3^{x^2} = 9 \cdot 3^{-x}$ think... $9 = 3^2$

$3^{x^2} = 3^2 \cdot 3^{-x}$

$3^{x^2} = 3^{2+(-x)}$ since the bases are = the exponents MUST be =

$\therefore x^2 = 2 - x$

$x^2 + x - 2 = 0$ factor this

$(x+2)(x-1) = 0 \longrightarrow$ this means either

$x + 2 = 0$ or $x - 1 = 0$

$\boxed{x = -2}$ $\boxed{x = 1}$

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2. c) $4^{\sqrt{x+1}} = 2^{3x-2}$ express 4 as $2^2$

$(2^2)^{\sqrt{x+1}} = 2^{3x-2}$ use law of exponents for powers

$2^{2\sqrt{x+1}} = 2^{3x-2}$ since bases are equal, the exponents must also be equal

$2\sqrt{x+1} = 3x-2$ bring the 2 <u>into</u> the radical

$\sqrt{4(x+1)} = 3x-2$ square both sides

$4(x+1) = (3x-2)^2$

$4x+4 = (3x-2)(3x-2)$

$4x+4 = 9x^2-6x-6x+4$

$4x+4 = 9x^2-12x+4$

$0 = 9x^2-12x+4-4x-4$

$0 = 9x^2-16x$

$0 = x(9x-16)$

$\boxed{\cancel{x=0}}$ or $9x-16=0$

extraneous

$9x = 16$

$\boxed{x = 16/9}$

CHECK: $2\sqrt{x+1} = 3x-2$

$2\sqrt{0+1} = 3(0)-2$

$2 \neq -2$

$2\sqrt{16/9+1} = 3(16/9)-2$

$2\sqrt{16/9+9/9} = 16/3-2$

$2\sqrt{25/9} = 16/3-6/3$

$2(5/3) = 10/3$

✓

2d) $4^{-|x+1|} = \frac{1}{16}$

$4^{-|x+1|} = 16^{-1}$

$4^{-|x+1|} = (4^2)^{-1}$

$4^{-|x+1|} = 4^{-2}$

$-|x+1| = -2$

$x+1 = 2$ or $x+1 = -2$

$\boxed{x = 1}$ $\boxed{x = -3}$

2e) $4^{-2x+1} = 8^{x-4}$

$(2^2)^{-2x+1} = (2^3)^{x-4}$

$2^{2(-2x+1)} = 2^{3(x-4)}$

$2^{-4x+2} = 2^{3x-12}$

$-4x+2 = 3x-12$

$12+2 = 3x+4x$

$14 = 7x$

$\boxed{x=2}$

2f) $9^{2x-1} = \left(\frac{1}{27}\right)^{x+2}$

$(3^2)^{2x-1} = (3^{-3})^{x+2}$

$3^{2(2x-1)} = 3^{-3(x+2)}$

$3^{4x-2} = 3^{-3x-6}$

$4x-2 = -3x-6$

$4x+3x = -6+2$

$7x = -4$

$\boxed{x = -4/7}$