Name: _____ Class: _____ Date: _____ ID: A

Pre-Calculus 12 - PRE TEST: Chapter 7 - Permutations and Combinations

Multiple Choice

Identify the choice that best completes the statement or answers the question.

C 1. A restaurant offers 4 different flavours of coffee, 3 different soup selections and 4 different sandwiches. In how many ways can a person select one item from each category (a coffee, a soup and a sandwich)?

a. 11
b. 18
c. 48
d. 54
e. 96

USE THE FUNDAMENTAL COUNTING PRINCIPLE

$4 \times 3 \times 4 = 48$

coffee soup sandwiches

b 2. Using the digits 2, 3, 6, 8, and 9, how many 3-digit whole numbers can be formed if repetitions are not permitted?

a. 14
b. 60
c. 120
d. 625
e. 1296

5 numbers to choose from

$5 \times 4 \times 3 = 60$

b 3. A multiple-choice test has 7 questions, with 4 possible answers for each question. If a student were to guess the answer to each question, how many different ways would there be to answer the test?

a. 28
b. 16384
c. 65536
d. 142926
e. 262144

7 questions $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^7$

b 4. A car licence plate consists of 7 characters. The first 4 characters are numerals from 0 to 9. The last 3 characters are letters excluding I, O, and X. How many different licence plates are possible?

a. 109
b. 121670000
c. 139827687
d. 161670000
e. 175760000

$10 \times 10 \times 10 \times 10 \times 23 \times 23 \times 23 = 10^4 \times 23^3$

C 5. How many permutations are there of all the letters in the word COMBINE?

a. 42
b. 49
c. 5040
d. 10080
e. 40320

7 letters with no repetition
∴ 7!

d 6. A teacher is assigning one each of the letter grades A, B, C +, C, C -, and F to the 6 students. If she assigned the grades randomly, in how many ways could these grades be assigned to the students?

a. 6
b. 12
c. 36
d. 720
e. 46656

6 × 5 × 4 × 3 × 2 × 1 = 720 ways

d 7. Calculate the number of ways that the "win", "place" and "show" horses (the first 3 finishers) can finish in a race with 9 horses.

a. 6
b. 27
c. 84
d. 504
e. 362880

9 × 8 × 7 = 504

a 8. Write the following expression without using the factorial symbol.

$$\frac{(n-4)!}{(n-1)!} = \frac{(n-4)!}{(n-1)(n-2)(n-3)(n-4)!} = \frac{1}{(n-1)(n-2)(n-3)}$$

a. $\frac{1}{(n-3)(n-2)(n-1)}$
b. $\frac{1}{(n-3)(n-2)}$
c. $(n-3)(n-2)(n-1)$
d. $(n-3)(n-2)$
e. $\frac{1}{(n-2)(n-1)}$

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b 9. Simplify. $\frac{12!}{3!2!4!2!}$ = $\frac{12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{6\times2\times4\times3\times2\times2}$

a. 415800
(b.) 831600
c. 207900
d. 1023
e. 35

= $11\times10\times9\times8\times7\times5\times3 = 831600$

a 10. Four different coins are tossed once each. How many ways can exactly 2 coins be heads and 2 coins be tails?

(a.) 6
b. 8
c. 16
d. 32
e. 64

$_4C_2 = 6$

or, in how many ways can the letters TTHH be arranged?

$\frac{4!}{2!\,2!} = 6$

H	H	T	T
H	T	H	T
H	T	T	H
T	H	T	H
T	H	H	T
T	T	H	H

C 11. On a 10 question multiple choice English 12 quiz, 3 answers are A, 2 answers are B, 2 answers are C, 1 answers is D, and 2 answers are E. How many different answer keys are possible?

a. 7560
b. 37800
(c.) 75600
d. 151200
e. 302400

$\frac{10!}{3!\,2!\,2!\,2!} = 75600$

C 12. In how many different ways can the letters of MISSISSIPPI be arranged?

a. 77
b. 3150
(c.) 34650
d. 48650
e. 69300

$\frac{11!}{4!\,4!\,2!} = 34650$

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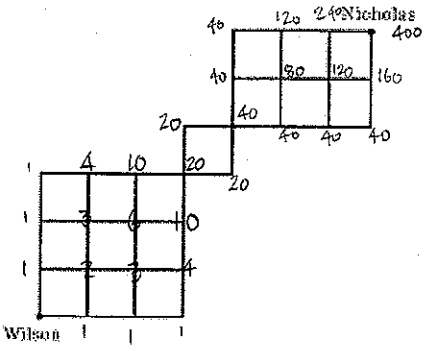
d 13. Wilson wants to visit Nicholas. Roads are shown as lines on the grid. Only north and east travel directions can be used. How many different paths can Wilson take to get to Nicholas?



$\frac{5!}{2!3!} = 10$

$\frac{2!}{1!1!} = 2$

$\frac{6!}{3!3!} = 20$

$10 \times 2 \times 20 = 400$

a. 20
b. 32
c. 200
(d.) 400
e. 600

e 14. How many different tests each with 15 questions can be constructed from a test bank containing 32 questions?

$_{32}C_{15} = 565\,722\,720$

a. 480
b. 496
c. 285182525
d. 471435600
(e.) 565722720

b 15. Write a simplified expression for ${}_nC_6$.

$= \frac{n!}{6!(n-6)!}$

a. $\frac{n!}{720}$

(b.) $\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{720}$

$= \frac{n \times (n-1)(n-2)(n-3)(n-4)(n-5)\cancel{(n-6)!}}{720\,\cancel{(n-6)!}}$

c. $\frac{n}{6}$

d. $n(n-1)(n-2)(n-3)(n-4)(n-5)$

e. $(n-6)!$

C 16. A committee of 5 people is to be chosen from a group of 12 people --- 6 women and 6 men. In how many ways can the committee be chosen so as to include exactly 3 men?

a. 15
b. 20
c. 300
d. 720
e. 924

exactly 3 men ⟶ ${}_6C_3$

∴ there must be

exactly 2 women ⟶ ${}_6C_2$

${}_6C_3 \times {}_6C_2 = 20 \times 15 = 300$ ways

C 17. From a deck of 52 cards, the 12 face cards and 4 aces are removed. From these 16 cards, 4 are chosen. How many combinations are possible that have at most 1 red card?

a. 126
b. 448
c. 518
d. 7842
e. 1302

AT MOST 1 RED CARD MEANS...

3 BLACK 1 RED and 4 BLACK 0 RED

${}_8C_3 \times {}_8C_1 + {}_8C_4 \times {}_8C_0$

$56 \times 8 + 70 \times 1 = 448 + 70 = 518$

C 18. Seven points are marked on a circle. How many quadrilaterals can be formed using any 4 of the 7 points?

a. 15
b. 20
c. 35
d. 70
e. 840

${}_7C_4 = 35$

C 19. What is the second number in the 80th row of Pascal's triangle?

a. 1
b. 78
c. 79
d. 80
e. 3081

80^{th} row contains ${}_{79}C_0$ ${}_{79}C_1$ etc...

2^{nd} number is ${}_{79}C_1$

$= 79$

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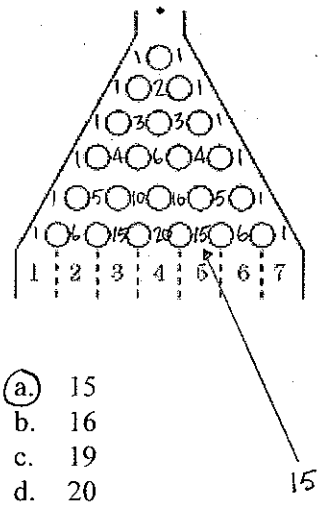
a 20. How many different paths are there that reach from the top to exit 5?



(a.) 15
b. 16
c. 19
d. 20
e. 32

e 21. Expand $(x-1)^8$ using Pascal's triangle.

$x^8 + 8(x)^7(-1) + 28(x)^6(-1)^2 + \ldots$

$x^8 - 8x^7 + 28x^6 - \ldots$

a. $x^8 + 8x^7 + 28x^6 + 56x^5 + 70x^4 + 56x^3 + 28x^2 + 8x + 1$
b. $x^8 + 8x^7 + 40x^6 + 56x^5 + 70x^4 + 56x^3 + 40x^2 + 8x + 1$
c. $x^8 + 8x^7 + 28x^6 + 56x^5 + 84x^4 + 56x^3 + 28x^2 + 8x + 1$
d. $x^8 - 8x^7 + 40x^6 - 56x^5 + 70x^4 - 56x^3 + 40x^2 - 8x + 1$
(e.) $x^8 - 8x^7 + 28x^6 - 56x^5 + 70x^4 - 56x^3 + 28x^2 - 8x + 1$

e 22. What are the fist four terms in the expansion of $(w-x)^{14}$

$w^{14} + {}_{14}C_1 (w)^{13}(-x) + {}_{14}C_2 (w)^{12}(-x)^2 + {}_{14}C_3 (w)^{11}(-x)^3$

$= w^{14} - 14w^{13}x + 91w^{12}x^2 - 364w^{11}x^3$

a. $w^{14} + 14w^{13}x + 91w^{12}x^2 + 168w^{11}x^3$
b. $w^{14} - 14w^{13}x + 182w^{12}x^2 - 364w^{11}x^3$
c. $w^{14} + 14w^{13}x + 91w^{12}x^2 + 364w^{11}x^3$
d. $w^{14} - 14w^{13}x + 91w^{12}x^2 - 168w^{11}x^3$
(e.) $w^{14} - 14w^{13}x + 91w^{12}x^2 - 364w^{11}x^3$

e 23. What is the seventh term in the expansion of $(b-5)^{13}$?

$t_{k+1} = {}_nC_k x^{n-k} y^k$

$k = 6$
$n = 13$
$x = b$
$y = -5$

a. $-134062500b^7$
b. $187687500b^7$
c. $134062500b^7$
d. $-26812500b^7$
(e.) $26812500b^7$

$t_7 = {}_{13}C_6 b^7 (-5)^6$

$= 1716 b^7 (15625)$

$= 26812500 b^7$

____ 24. What is the middle term in the expansion of $(r-5)^6$?

The expansion of $(r-5)^6$ has 7 terms. Therefore we are looking for the 4th term. Use $t_{k+1} = {}_nC_k x^{n-k} y^k$ where $k=3$, $n=6$, $x=r$, $y=-5$

a. $-1875r^3$
b. $2500r^3$
(c.) $-2500r^3$
d. $-1875r^4$
e. $9375r^4$

$t_4 = {}_6C_3\, r^3(-5)^3$
$= 20(r)^3(-125)$
$= -2500r^3$

Short Answer

25. Without using a calculator, clearly show your evaluation of $\frac{100!98!}{99!97!}$

$\frac{100 \times 99! \times 98 \times 97!}{99!\ 97!} = 9800$

26. How many ways can 3 boys and 3 girls sit in a row if all the boys sit together, and all the girls sit together?

3B 3G

2 ways for Boys and GIRLS

3G 3B

but, each boy and girl can sit 3! ways

∴ 3! 3! × 2 ways = 72 ways

27. Calculate the number of different 5-card poker hands that make a full house - 3 of a kind plus a pair.

POKER HAND ⟶ 3 of a kind ${}_4C_3$ ⟶ picking the number ${}_{13}C_1$

2 of a kind ${}_4C_2$ ⟶ ${}_{12}C_1$

∴ # of different poker hands that make a full house

$= {}_4C_3 \times {}_{13}C_1 \times {}_4C_2 \times {}_{12}C_1 = \underline{\underline{3744}}$

12 other numbers are left to be chosen after the first three cards are taken.

28. The 7th term of $(2x-1)^n$ is $112x^2$. Determine n.

Use $t_{k+1} = {}_nC_k x^{n-k} y^k$

in this case $x = 2x$

$y = -1$

$k = 6$

$t_7 = 112x^2$

∴ $112x^2 = {}_nC_6 (2x)^{n-6} (-1)^6$ but, $(-1)^6 = 1$

$112x^2 = {}_nC_6 (2x)^{n-6}$

the "x^2" comes from $(2x)^{n-6}$

∴ $n-6=2 \longrightarrow n=8$

CHECK: ${}_8C_6 (2x)^2 = 28(4x^2) = 112x^2$